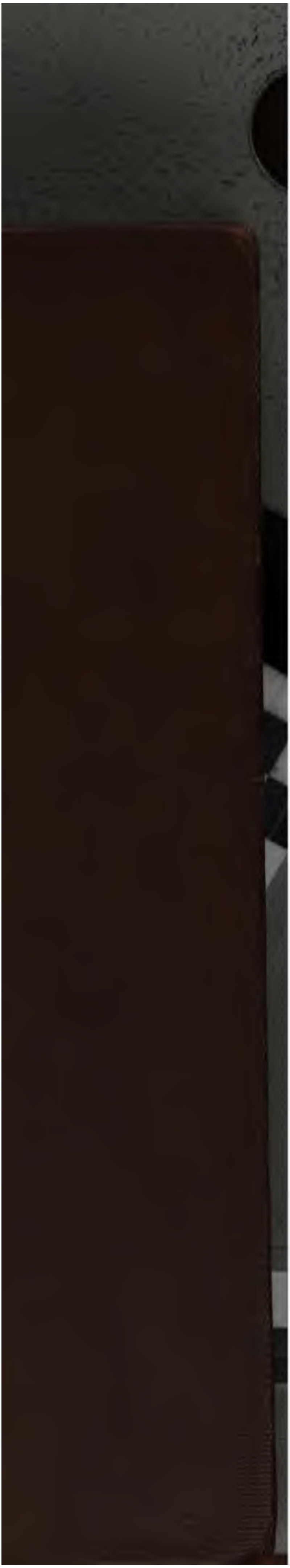

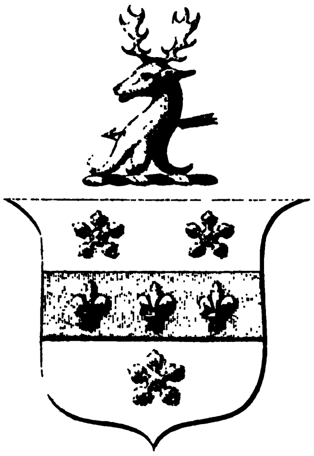

Joshua Bates.

Farrar

AN

EXPERIMENTAL TREATISE

ON

OPTICS,

COMPREHENDING

THE LEADING PRINCIPLES OF THE SCIENCE,

AND

AN EXPLANATION OF THE MORE IMPORTANT

AND CURIOUS OPTICAL INSTRUMENTS AND OPTICAL PHENOMENA,

BEING

THE THIRD PART

OF

A COURSE OF NATURAL PHILOSOPHY,

COMPILED

FOR THE USE OF THE STUDENTS OF THE UNIVERSITY

AT

CAMBRIDGE, NEW ENGLAND.

BY JOHN FARRAR,

PROFESSOR OF MATHEMATICS AND NATURAL PHILOSOPHY.

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The body of this volume, like that of the preceding, was selected from Biot's *Precis Elementaire de Physique Experimental.*

In an elementary course of Natural Philosophy the space alotted to Optics is too limited to admit of any thing like a complete treatise. The plan adopted by Biot in his smaller work has been highly approved, and seemed, on the whole, to be best suited to the state of information and wants of most learners. Reference is frequently made to the larger work of this distinguished philosopher for the algebraic formulas and more detailed information relating to the subject under discussion. The part upon the polarisation of light, amounting in the original to about one hundred and fifty pages, is omitted, as not comporting with the design and limits of the work, already perhaps too extended for the time appropriated to these studies. A brief account is given in a note of this new branch of Optics, drawn up by Biot himself, and appended to his translation of Fischer's *Physique Mécanique.*† The other notes are intended to furnish information upon several other topics that have not found a place in the text.

The parts of this course of Natural Philosophy already published are, a treatise upon Mechanics, and a trea-

† The original not being at hand, the compiler has made use of a translation, contained in Coddington's Optics.

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ELEMENTARY TREATISE

ON

OPTICS.

General Remarks.

1. When the sun first appears above the horizon and presents itself suddenly to our view, we feel persuaded that there must be some mode of communication between this luminary and ourselves, which informs us of its existence without the necessity of our coming in contact with it. This mode of communication, which thus takes place at a distance, and through the medium of the sight, constitutes what is called *light*. Bodies, which are capable of exciting it directly, and of thus making themselves known to us, are called self-luminous, as the sun and stars. It would seem, indeed, that all material substances become self-luminous when their temperature is sufficiently raised, and they lose this property by being deprived of their heat. If they receive light, however, from a luminous body, after they have ceased to be self-luminous, they are still capable of sending to us the light thus received, as if it were their own, and in this case they become visible by *reflection*. In this way we perceive the objects about us while the sun is above the horizon, and all becomes obscure and invisible when this light is withdrawn.

In all cases, when an object transmits to us a sensation of its existence by means of light, this transmission takes place in a right line; for if we place fine threads of silk or metal parallel to each other and in the same plane, a luminous point situated at some distance beyond the threads in the same plane will be eclipsed by them; but if we move it a little out of the direction of this plane, it will become visible. Moreover, if we take two metallic plates perfectly plane and bring them by degrees to-

wards each other, by directing the sight to the light of the sky through the space which separates them, we shall always be able to perceive it, however small the distance between the plates; but when one plate is concave and the other convex, vision ceases before they are brought into contact. This fact is confirmed by daily observation; and as it is of continual use, it will be proper to give it a simple enunciation. For this purpose, every right line, drawn from any point of a luminous body to the eye, has a particular designation, and is called a *luminous ray;* and direct vision is supposed to follow the course of these rays. This definition may likewise be employed in a physical sense, to distinguish the component parts of a luminious collection, at least as far as our senses are capable of advancing in the process of dividing. For, however fine a ray of light may be, as long as it is sensible to our organs, we find in it identically the same properties as in the largest collection.

2. It is proved by astronomical observations that the communications established by light between luminous bodies and ourselves is not instantaneous. When the sun is at any point of its orbit, we do not perceive its presence at this point until about 8′ 13″ after it has arrived there. When the satellites of Jupiter, which are small moons illumined by the sun, after having been eclipsed by the body of their planet, disengage themselves from its shadow, a certain time elapses between the absolute instant at which they are thus disengaged, and that at which we begin to perceive them. The interval is longer or shorter, as the earth is more or less distant from Jupiter, and is exactly proportional to this distance. Hence we conclude that the velocity of light is strictly uniform throughout the whole extent embraced by the earth and Jupiter in their annual revolutions.

It follows, moreover, from these phenomena, that after the absolute instant when the satellites of Jupiter enter into the shadow of this planet, we still see them without it, since the sensation which we have of them results from their previous presence at a point of their orbit which they occupied a few minutes before; and also when they disappear, their light has already been interrupted for some time. Thus the communication resulting from the presence of these bodies at any point continues to take place even after they have left it. This communication then must be effected either by means of impulses propagated

through an elastic fluid from luminous bodies to us, as sound is transmitted through the air, or by an actual emanation of material particles from these bodies. In all cases, as the sensation of vision takes place even through the substance of certain bodies, called *transparent* or *diaphanous*, it is necessary either that the impulse of the elastic fluid should continue to be propagated through the pores of these substances, or that the luminous particles should continue to move within them and be capable of passing through them.

3. Each of these opinions has had its defenders. Those who are inclined to favour the supposition of an elastic fluid, dwell upon the facility with which this theory adapts itself to rapid and uniform transmission. They insist upon the improbability of an actual emanation of particles having the velocity belonging to the particles of light, and at the same time, of such extreme tenuity as to be able to pass readily through transparent bodies. As to this point we must be guided solely by facts; for *slow* and *rapid* are nothing in themselves, any more than *great* and *small*. The motion of a cannon-ball to us is so rapid that our eyes cannot follow it; this, however, is very slow compared with the revolution of the earth upon its axis; and this in like manner is slow compared with the earth's annual motion; indeed the earth's annual motion falls far short of the rapidity with which light is transmitted. It is certainly more difficult for us to give a large body a great velocity than a small one, since our strength is limited; but what community or what comparison is there between our limited powers, and the extent and kind of power which operates in nature? None, absolutely none. Accordingly, if we divest ourselves of this prejudice, and examine the phenomena by themselves, we shall perceive that a very great part of them take place in a manner exactly conformable to the idea of an emission. When light passes through transparent bodies its motion is precisely such as it ought to be, were it composed of a substance, capable of being attracted by these bodies. If we observe its motion in gaseous or liquid substances of different natures, and then mix these substances, (which we suppose to be such that they can have no chemical action upon each other) the motion of light through the compound may still be calculated by the laws of the attractions of the component substances, pre-

viously known; and the result of this calculation agrees exactly with observation. But who can say in what manner undulations must be formed? And without being able to answer this question, it would seem that they must be formed according to extremely complicated laws. In fine, it is known from other phenomena that luminous rays may be so modified and prepared that their different sides shall present different physical properties; a fact which agrees very well with a series of particles, but which it is much more difficult to conceive and represent, if we suppose a series of impulses. Moreover, these properties, relating to the sides of the rays, are so inherent in them that they manifest themselves in passing through liquids perfectly homogeneous, as the essence of citron, for instance, and that of turpentine; and, if the rays penetrate these liquids in a direction perpendicular to their free surface, which seems to give to pulsations all the conditions of perfect symmetry, they continually exhibit during their transmission properties not symmetrical about the perpendicular to the face of incidence; this it would be very difficult to comprehend on the supposition of transmitted impulses, but is very easily understood if we admit the theory of luminous particles having different properties on their different faces, and being successively submitted to the actions of the particles of a homogeneous medium through which they pass.

In what I shall hereafter offer on the phenomena of light, I shall employ this theory which supposes it material, and which affords in almost all cases great facility of conception and representation; then, when we have studied the phenomena in this manner, I shall present the views that must be substituted according to the system of undulations, and apply them to some phenomena, for which this system offers an easy explanation, while we have never yet been able to deduce them from the hypothesis of the materiality of light.

4. In this exposition we have considered vision as propagated from the object to the eye, but it is not on the external surface of the organ that the sensation takes place; it is in the interior, and according to common opinion, on a nervous membrane which covers the back part of the eye, called the *retina*. Indeed, when the different media, composing the other parts of the organ, happen to be hardened or affected by disease, so as to prevent the light from reaching the retina, the power of vision is destroyed;

but it is again restored when the parts of the organ which have become opaque are removed. Each of these parts may be separately removed without entirely destroying the sight; but if the retina is affected, the power of vision is lost forever. We know, moreover, that two large nerves, proceeding from the brain, spreading over the posterior surface, and branching out into an infinite number of ramifications terminate in the retina, or to speak more correctly, the retina itself is only an expansion of these nerves. Now we know that in all the other organs, sensation is propagated by the nerves. This analogy confirms the supposition that the sensation of vision takes place immediately at the retina, either by impulses propagated through an elastic fluid, if light be transmitted by undulations, or by the direct impression of luminous globules if light is material. I shall treat more particularly of the internal structure of the eye, after having made known the theory of optical instruments; for the eye itself is an optical instrument so perfect and admirable, that the most profound examination fails of comprehending all its wonderful properties, and the most exquisite art has not succeeded in imitating it. Thus far these preliminary observations will suffice; we shall confine ourselves to the consideration of the back of the eye or the retina, as the centre of the organ of sight.

5. When we view an object sufficiently great to be perceptible, the rays coming from its opposite extremities, arrive at the eye in different directions, and consequently cross each other so as to form a certain angle at their point of incidence before the pupil. This is called the *visual angle* or the *apparent diameter* of an object, since, in fact, as we shall see hereafter by many examples, we judge of the actual bulk by the size of the visual angle which it subtends in the eye, together with the idea we have of the distance at which we suppose it to be placed.

When light passes from a luminous body to us, it always comes through different media, such as air, water, or other transparent bodies, whch afford it a passage more or less free. The rays upon entering these bodies, sometimes pursue their course in a straight line; but most commonly they are turned from their original direction, and this phenomenon is called *refraction.* Besides this cause of deviation, it frequently happens that light meets with smooth surfaces which send it off or *reflect* it, by which objects are presented to our sight in an indirect way,

when we happen to be in the direction of the rays thus reflected. We shall study the refraction and reflection of light in succession. It would seem proper to begin with the first of these classes of phenomena, since, as we ourselves are immersed in a material fluid, the air, vision cannot take place without the action of the air upon the luminous rays; but as this action is very feeble, and hardly turns them from their natural direction, we shall defer this subject, and begin by investigating the laws of the phenomena of reflection, which are much more simple, and which, taken together, constitute the first branch of the science of optics, called *catoptrics*.

CATOPTRICS.

General Laws of Reflection.

6. In order that the surface of a body may reflect the light regularly and give a distinct image of the luminous points by means of which it is rendered visible, it must be carefully polished, that is, its little inequalities must be removed, as far as possible, by friction. Such is the state to which glass, crystals, and metals, are reduced by art. We shall first consider the phenomena presented by surfaces thus prepared; and afterwards endeavour to show in what manner the polishing contributes to the effect in question.

To proceed methodically in the study of the laws of reflection, we shall begin by determining what these laws are with respect to plane surfaces. It will be easy to extend them, when thus determined, to curved surfaces. For, in all the modifications which light undergoes, by the action of bodies of a sensible extent, the luminous rays may be considered, (at least, as far as they are perceptible to our senses) as mathematical straight lines; so that at each point of a curved surface, reflection takes place in exactly the same manner as if the ray fell upon a plane touching this point; and since we may always calculate the position of the tangent plane for all the points of a given surface, it follows that the reflection of light from any curved surface whatever will be a subject of pure calculation, when the laws according to which it takes place from plane surfaces are once known.

7. For the purposes of this inquiry, and with respect to optical experiments in general, it is indispensable that we should be provided with a room which receives the direct rays of the

sun during a part of the day, at least, and which is furnished with window-shutters made very tight, so that it may at any time be rendered completely dark. In one of these shutters an aperture is supposed to be made of any convenient size, and covered with a metallic plate, pierced with several holes of unequal diameters, and capable of being opened or shut at pleasure. One of these holes being opened when the rays of the sun fall directly upon the shutter, the light will enter the room in the form of a beam or fine collection of rays, which become sensible by being reflected from the particles of dust floating in the atmosphere. We shall hereafter make known a more convenient and ingenious apparatus; the one just described will suffice for the present.

Now if this beam be made to fall obliquely upon a polished, transparent plate of glass, placed horizontally, the following appearances will be exhibited;

(1). The luminous beam is not wholly transmitted through the surface. A part is reflected upward, in a direction depending on its obliquity. If the eye be placed any where in this direction, a lively, brilliant image of the sun will be visible, which seems to come from beneath the glass in the direction of the floor.

(2.) The point where the beam of light meets the plate is visible from every part of the room; but when thus viewed no regular image of the sun is presented, and the light is incomparably less brilliant than when the eye of the observer is placed in the direction of the reflected beam.

(3.) A portion of the incident light escapes reflection at the first surface of the plate and passes to the interior. Arriving at the second surface, another partial reflection takes place, and the remainder passes into the air beneath the glass.

If we now consider the phenomena presented by the first surface, we shall observe that there are three distinct operations. A part of the incident light is reflected regularly in a particular direction; another part is reflected indifferently in all directions, as if the body were not polished; and finally the remainder is transmitted without being reflected. To distinguish these two modes of reflection which take place in the same body, I shall call the first *specular reflection*, since it is this which gives a regular image, whatever be the reflecting surface. I

shall call the other mode *radiant reflection*, because it scatters the light in all directions about the point of incidence, as if this point had by itself a radiating power.

If we substitute for the plate of glass one of polished metal, the two first only of these phenomena will be observed. The polished metal reflects regularly one portion of the incident light, disperses another portion, and absorbs or extinguishes the rest; this remainder answers to what the transparent body transmits.

By confining ourselves, therefore, for the present to the two first phenomena which constitute reflection, we shall in the first place inquire into this dissemination of light which renders the point of incidence visible from all parts of the dark room. If we repeat the experiment with different reflecting bodies, and with the same body having different degrees of polish, we shall soon perceive that the imperfection of the polish is the determining cause of the phenomenon. For, the greater this imperfection is, the nature of the reflecting body remaining the same, the more considerable is the portion of light thus dispersed, and the more feeble, on the contrary, is the specular reflection. To be convinced of this, we need only take a plate of glass, having a plane but unpolished surface, and to present it successively, under different angles of obliquity to a solar ray, introduced into a dark room. When the ray meets the plate in a direction nearly perpendicular to its surface, the portion of light reflected specularly will be insensible; the radiant portion, on the contrary, will be very strong, and will render the point of incidence dazzling; but by inclining the reflecting surface to the ray, we shall find that this portion becomes less, and the specular reflection beginning to take place, gives at first a feeble reddish light; soon this light increases, and finally, when the ray becomes nearly parallel to the surface of the plate, it will be almost as strong and as white, as it would be from glass highly polished. Analogous results are observed, if instead of examining the reflection of a solar ray, we undertake to observe, by reflection from an unpolished plate, the images of very bright objects, for example, that of a building illuminated by the sun; for while the incident rays make large angles with the unpolished surface, the images of the objects are not distinctly formed. But soon they will

begin to appear when the rays begin to be inclined to the reflecting surface; and in the end they will become perfectly bright and distinct with the smallest inclinations.

Now the direction of the incident ray in relation to the reflecting surface being given, let us inquire what will be the direction Fig. 1. of that part of the light which is reflected specularly. For this purpose we may make use of the instrument represented in figure 1, which we shall hereafter have occasion to use. It consists, in the first place, of a circular plane AZB, placed vertically upon a firm stand capable of being levelled. The circumference AZB is graduated, and carries too metallic indices S, O, turning on the same centre and having two small holes S', O', at equal distances from the plane of the circle. At the centre C is placed a plate of polished glass, CG, which, by means of screws, is fixed perpendicularly to the plane of the circle; it consequently takes a horizontal position when the circle is vertical. In order to give it this adjustment the plane of the circle is first rendered vertical by being directed to the vertical bars of a window frame or other vertical object; a spirit level is then placed upon the glass plate, and the plate is moved by means of its adjusting screws till this level indicates a horizontal position. Finally, above this glass and before the centre of the circle is fixed a metallic plate, or thin wedge, the rectilineal edge of which CL forms a straight line, proceeding from the centre C perpendicularly to the plane of the circle; and upon this line is made a light stroke C', at the same distance from this plane with the hole in the indices; so that the three points, S, O, C', are always in the same plane parallel to the graduated circle. The instrument is now to be placed before an open window, so that the light from abroad may enter the hole S, and fall upon the glass under different angles. The index O is moved backward or forward till the image of the hole S is seen through the opening at O exactly upon the edge CL of the central plate. It is shown by experiment that this is possible; and the point of incidence is always found to fall precisely upon the stroke C', therefore, *the incident ray and the reflected ray are comprehended in the same plane perpendicular to the reflecting surface.* This is the first fundamental law of reflection. Moreover, the two rays meeting at the axis CC' of the graduated circle, their respective inclina-

tions to the reflecting surface are measured by the arcs BS, AO, which may be read off upon the graduated circle, the divisions commencing at the horizontal diameter AB. This being done, it will be found that for all possible positions of the indices, *the incident and reflected rays make the same angle with the reflecting surface.* This is the second general law of reflection, and being taken in connexion with the preceding, it determines all the circumstances of this phenomenon.

8. Through the point of incidence C' suppose a line $C'Z'$, drawn perpendicularly to the reflecting surface; the angle $S'C'Z'$, formed with this perpendicular by the incident ray is commonly called the *angle of incidence*, or simply *the incidence*, and $O'C'Z'$ is called the *angle of reflection.* The second general law of reflection, therefore, deduced from the preceding observations, is, that *the angle of reflection is always equal to the angle of incidence.*

Of the Plane Mirror.

9. The laws of reflection being known, it is easy to deduce from them the appearances which are observed when the reflecting surface is a plane.

Let S be a radiant point, O the eye, and AB the reflecting plane, which I shall suppose for the present of indefinite extent. Fig. 2. Among all the luminous rays which proceed from S, there will be one, as SI, which, after being reflected, will go to meet the eye at O, in the direction IO. Then the angles SIA, OIB, will be equal, according to the second law of reflection. Now draw from the point S, a perpendicular SA, meeting the reflecting surface in A, and produce this line on the opposite side of the mirror to D, making AD equal to AS. From the point D draw the straight line DO to the eye; DO, it is manifest, will be the direction of the reflected ray, and the point S, where it cuts the surface of the mirror, will be the point of incidence. Moreover, if the luminous object and the eye are considered as mathematical points, without sensible extent, the ray, thus determined, is the only one which can be reflected to the eye. But the pupil or opening through which the light is admitted into the eye is

not a mathematical point, but a circle of sensible extent which
Fig. 3. in man is about 0,08 of an inch in diameter, and which may be represented by LL. All the reflected rays which can enter this aperture will reach the retina and contribute to vision. Now each of these rays is determined by the construction just given; whence it is evident that they will form a *pencil* or cone with a circular base, the vertex of which will be D, and the base LL. It is a fact, that when the eye estimates freely the distances of luminous points, it supposes them placed at the point from which the rays that enter the eye diverge. Thus to the eye situated at O, the luminous point, seen by reflection, will appear at the point D, that is, as far behind the mirror as it really is before it.

If the radiant object has a definite extent, each of the radiant points, presented by it, will produce its own particular image, according to the laws which we have just explained, and the assemblage of these images will compose the image of the object.
Fig. 4. Suppose, for example, that this object is an arrow SS'; the point S of the arrow will have its image at D, the image of the other extremity will appear at D', and the intermediate points will be seen in the straight line DD'. Thus the entire image will be comprehended between the extreme reflected pencils DO, $D'O$; its absolute magnitude DD' will be equal to that of the arrow itself; but it will appear inverted from right to left.

According to what we have just now said respecting figure 3, each point of this image is regarded by the eye as situated where it is actually fixed by our geometrical construction. If we substitute, therefore, in the place of this image a real and similar object, that is, an arrow in every respect equal to SS', placing its point at D, and its base at D', and if we look at it from the point O, the mirror being removed, we shall see it precisely as we see the image DD', having the same magnitude, the same visual angles, and the same degree of brightness, since all the luminous points which compose the real object would transmit to the eye cones of luminous rays exactly similar to those which seem to proceed from the image. It hence follows that objects are seen in the plane mirror with the same forms, the same brightness, and at the same distance, as they would be if viewed through the medium of the air, provided that the quantity of light is the same in the two cases; for we must make

allowance for the portion of light which is transmitted when the mirror is transparent, or absorbed when it is opaque.

10. What has now been said is sufficient for the resolution of all problems which may occur respecting reflection and vision by plane mirrors.

As the reflection of light takes place rigorously according to the law we have just demonstrated, it may be employed with great advantage in measuring the angles formed by two plane and polished surfaces. As this measurement is necessary in a great number of physical inquiries, and as no other method has ever been employed sufficiently nice and accurate for this purpose, I shall give some simple examples of the process.

When we operate upon bodies having faces of a sensible extent, for example, on prisms cut and polished by art, like those which are employed in optical experiments, we make use of the instrument which has already been applied to determine the laws of reflection. For this purpose, instead of bringing the glass plate *GG* in contact with the edge *CL*, it should be placed at the distance of about $\frac{1}{10}$ of an inch, being always made perpendicular to the plane of the circle, by means of the adjusting screws with which the instrument is provided. In order to give it this position without other aid than that which the instrument affords, we move the index *O* nearly to the highest part of the circle, and placing the eye behind it, we regulate the glass so that the reflected image of the hole *O'* and that of the eye shall return through this hole from the fixed mark *C'* upon the edge *CL*. When this takes place we are sure that the ray *OC*, parallel to the circle, is at the same time perpendicular to the glass, and reciprocally that the glass is perpendicular to the plane of the circle. We then place upon the glass one of the faces of the prism whose angles we wish to measure. The edge of the prism is now made to slide under *CL*, by turning it in such a manner that the upper surface shall also be perpendicular to the plane of the circle; and this condition is fulfilled when, the index *S* being placed in any point of the circumference, its image, as seen through the other index, moved to a corresponding point, appears on the mark *C'*. Let *S* and *O* be the positions of the two indices answering to this condition. Then it is evident that *N*, the centre of the arc *SO* (a point easily determined by means of the divisions upon the arc) is in the direction of the line rais- Fig. 5.

ed from the centre C perpendicularly to the reflecting surface. Moreover, CZ is perpendicular to the other face; therefore the angle ZCN, measured by the arc ZN, is the angle comprehended between the two surfaces of the prism, and which it was proposed to determine. This ingenious method, which was invented by M. Cauchoix, requires but little time and is very accurate in its results.

11. The same principle differently applied, has led to a great number of instruments analogous to this, which are called *goniometers*, that is, instruments for measuring angles. They will be easily understood after what has now been said, for the means of observation and verification are essentially the same in all. I shall nevertheless describe that invented by Dr Wollaston, because it is particularly applicable to mineralogy.

This instrument, represented by figure 7, is composed of a vertical circle of copper, graduated on its edge, and turning about a horizontal axis AA. It is itself supported upon a vertical foot CP. Within the axis AA, made hollow for the purpose, turns the axis $a\,a$, the projecting extremity of which carries several pieces having rectangular movements, upon which are fixed, with pincers or wax, the crystal whose angles are to be measured. To make use of this goniometer, we stand before a building having several horizontal lines parallel to each other; we then place the base of the instrument on some horizontal plane, so that the limb shall become vertical and perpendicular, or nearly perpendicular, to the lines which are to be employed as sights. The first condition is easily fulfilled by arranging the plane of the limb along some of the vertical lines presented by the building. This being done, we place the eye very near the crystal, and looking at the building by reflection from one of its faces, we turn this in such a manner, that one of the horizontal lines the most elevated, thus seen, shall coincide with one of the inferior lines seen directly. I shall mention soon how we may arrive at this condition. When we have obtained it, we turn the interior axis $a\,a$, until the same coincidence shall be observed likewise on the other face, whose inclination to the first we wish to measure. We arrive at this by several trials. Now when this coincidence can be thus obtained successively upon the two faces without changing the place of the eye, without touching the crystal, and by the rotation merely of

the axis *a a*, we are sure that the intersection of the two surfaces is exactly horizontal, and consequently parallel to the axis *a a*. Then we do not touch the crystal, but setting out from one of its positions, in which the reflection is observed on one of the two surfaces, we turn the limb until the reflexion and the coincidence are observed likewise on the other. This motion takes place by means of the great axis *AA*, which turns with the axis *a a*, the crystal and the limb. The arc by which this has turned, is measured by the division traced upon the limb, and it is evidently equal to the supplement of the angle formed by the two surfaces. But the division traced upon the limb is written in a manner to indicate the angle itself, at least when we first put the index on the point zero.

In order that the application of this method may be easy and sure, it is necessary that the dimensions of the crystal and its distance from the eye should be infinitely small, compared with the distance of the objects used as sights. For if this is observed, the fixed position of the eye is no longer a necessary condition any more than it is in observations made at sea with instruments of reflection. Thus by placing the eye very near the crystal, the approximation of the lines of sight may be considered as indefinite. Dr Wollaston was accustomed to place the instrument in a chamber at some distance from the window, the bars of which were used as sights, and of which the upright parts served to place the instrument in a vertical plane. But without the address of this practised observer, sights so near could not be safely employed; for the liability to error increases with their nearness. We may in general make use of the horizontal and vertical lines of an edifice sixty or eighty yards distant; then, to render the first face of the crystal perpendicular to the limb, we first direct the shaft *t o* parallel to its surface; then, Fig. 8. without taking it from this direction, we turn it upon its axis until the reflected image of one of the horizontal lines becomes parallel to the direct image, and a coincidence may be effected by the mere motion of the axis *a a*. We then see if the same condition is fulfilled for the other face of the crystal, and as in general we find it is not, we effect it by turning the branch *b c* about the point *c* without touching the shaft *t o*; this motion being perpendicular to the plane of the limb cannot alter the perpendicularity of the first face; but for greater security we turn the

which is not known to us; and such is the nature of the understanding, that we cannot without this aid, follow the connexion of a series of facts, the principle of which is not discovered. It is thus that the astronomer, who is unable to comprehend generally in his formulas the course of a heavenly body, when it is very complicated, calculates successively the different parts of its path adapted to different orbits, which are not respectively applicable, except through a small extent, and which are changed according as it is found that they become erroneous.

When we consider light as an actual emission from the luminous body, the phenomena of specular reflection seem, at first sight, to be simple results of the elasticity, which causes the luminous particles to be reflected from the surface of polished bodies, as an ivory ball rebounds from a marble table, making the angle of reflection equal to the angle of incidence. But this idea which first presents itself, and which was also first adopted, will not bear examination.

22. Without being acquainted with the absolute dimensions of the particles of light, it will be readily understood that they must be exceedingly small, so small that the most powerful microscopes cannot magnify them to a perceptible size; if it were otherwise, how could they pass, as they do, through large masses of glass, water, and other transparent substances, not only without any retardation, but, as we shall presently show, with an accelerated motion. And finally, when with their incredible velocity, they fall by millions every instant upon the delicate membranes of the eye, how happens it that the organ is not torn in pieces, and that we do not suffer a thousand pangs, unless it be true that the particles are so minute as to render their impulse almost insensible? Now I ask what proportion there can be between the particles of light and the inequalities which still remain upon bodies, polished by processes of art? And considered in relation to light, what difference is there between bodies polished and unpolished? For in polishing bodies we only rub them, as it were, with small and hard particles of dust, which indeed remove their greater inequalities, but leave them furrowed in every direction. But these particles of dust, which we can recognise with a microscope and even discover with the naked eye, are vast masses, and the furrows which they leave upon bodies are of immense depth, compared with the particles of

light. If light, therefore, coming in actual contact with the surface of bodies, were reflected by the mere force of elasticity, the little particles of which it is composed would be dispersed in every direction by the elevations, or lost in the deep cavities, of these bodies; and reflection from the most carefully polished would be hardly more perfect than that from the most rough. But, since reflection from polished bodies is much more abundant, more perfect and regular, we infer that it is not the mere mechanical effect of elasticity, and that the particles of light do not come in actual contact with the polished surface.

23. The force by which the rays are repelled, acts therefore at a distance from the solid surface. It acts, moreover, in general, unequally upon the different particles of the same ray. For in most cases, in which reflection takes place, one part of the incident light is reflected and another transmitted, either because the repulsive force is variable in its action, being at one moment more active and feeble than at another; or, as seems most probable, because all the luminous particles which follow each other successively in the same ray, are not, at the moment of their incidence, in the same physical state, and equally susceptible of being repelled.

24. As to the nature of the reflecting force, we are entirely unacquainted with it. We do not know whether it belongs to the particles of the reflecting body or to those of light; whether it acts by repulsion or attraction; and, considering only its general effects, we might represent it by numberless mechanical causes. But without attempting to determine its nature, we may always compare it to a repulsive force acting at the points of incidence, and tending to repel a certain number of the particles which compose the incident rays.

Let us suppose that the waving line AB represents the plane Fig. 21.
surface of a body covered with natural asperities, or those which art cannot remove, and let us imagine that all the points of this surface, or more generally, that all the particles of the two contiguous media which compose it, exert at a certain distance a repulsive force upon the luminous particles which approach it. This force must be very powerful at the distance where reflection takes place, since it is then sufficient to destroy the prodigious velocity with which the particles of light are impelled, and

to turn them back in the opposite direction; but it must diminish rapidly as the distance increases. For, if it were otherwise, the direction of the reflected rays would be affected by the parts of the reflecting body, which are at a sensible distance from the point of incidence, and then this direction would depend upon the general form of the reflecting surface, whereas it is determined solely by the direction of the superficial element, which the ray strikes upon; and it really takes place as if all the rest of the reflecting surface did not exist. Moreover, if the thickness of the reflecting body be gradually reduced by grinding away the second surface $A'B'$, without altering the first, the regularity and amount of the reflection are not at all effected, at least till the body is brought to an extreme degree of thinness, scarcely attainable by art. Thus the particles situated at a greater depth than this limit, cannot extend their influence to the reflecting surface, or at least to the distance from it at which reflection takes place; and since their force, which is so great at very small distances, is diminished to such a degree as to become insensible at a little depth, it follows that it decreases as the distance varies with very great rapidity. This will, therefore, be one of the characters which we ought to recognise.

25. Let us now suppose that a beam of parallel rays of light
Fig. 22. SM, $S'M'$, falls at any angle upon the reflecting surface AB of indefinite extent, and let us consider what takes place with respect to the luminous particles M, M', when they are near enough to begin to feel the repulsive action of the particles of the body. If the surface is perfectly plane, as AB, or which is the same thing, if its inequalities are insensible compared with the distance to which the repulsive force extends, the activity of this force will be the same at every point of the surface, and consequently, the effect upon all the particles of light M, M', whose directions, velocities, and dispositions are the same, will be equal. This is the case with polished bodies; but if the
Fig. 23. reflecting surface is broken up with large elevations E, E', E'', separated by deep cavities, F, F', the reflecting force cannot be equal at all these points. It is evident, for example, that the luminous particles which enter the cavities will not be reflected in the same direction with those which fall upon the inclined sides of the elevations, nor those which fall upon the sides like those which fall upon the summits. It may even happen that

those which enter the cavities are not reflected from them at all, being driven downward by the repulsion from the elevations. Such a surface can only produce a weak and irregular reflection, like that which takes place at the surface of unpolished bodies; nor is it necessary that the inequalities of the surface should be so large as to be perceptible to the touch or to project sensible shadows from the one to the other, in order to produce such an effect; it is sufficient that they are of sensible dimensions, compared with the distance to which the reflecting force extends. Of this kind are the inequalities of plane glass which has not yet been polished, It is plane if we consider only the general direction of its surface; we can neither measure the height of its asperities, nor obtain points sufficiently small to be introduced into its cavities; but these inequalities are too great for light, and the oppositions resulting from them in the direction of the repelling force, weaken the general repulsion while it makes it irregular. To remedy this inconvenience we endeavour to remove, or at least to diminish, these asperities by rubbing the surface of the glass with some substance whose own asperities are easily overcome, as paper or taffeta, stretched and made smooth by friction; but we should effect the same purpose by diminishing the velocity of the luminous particles, which the repulsive force must overcome before reflection can take place. Now this we do in fact by rendering the direction of the rays more oblique to the reflecting surface, that is, by causing them to form a less angle with its direction. For, if we conceive the velocity of the incident particle of light to be decomposed into two others in rectangular directions, of which one is parallel to the reflecting surface, and the other perpendicular to it, it is evident that only the latter will require to be overcome by the repulsive force of the surface; and it is also evident, that this will diminish as the incident ray becomes more oblique. But besides this, the obliquity is favourable to reflection in another way; for the luminous particles penetrate less directly into the cavities at the surface of the reflecting body, and are more exposed to the action of the summits of its elevations, which form a surface sensibly plane (for to this state we suppose the body reduced,) and produce a uniform repulsive force throughout the whole extent of its superficies. Indeed, we find by experiment that reflection is nearly the same from polished and unpolished glass, when the luminous

which serves to measure refraction, and we observe the deviation of the luminous rays as in a solid prism, regarding only those which pass through the cavity in which the liquid is enclosed.

Determination of the Ratio of Refraction in Aeriform Substances.

39. The refraction of the gases is observed in the same way as that of liquids, by introducing them into prismatic vessels, the faces of which are closed by parallel plates of glass; but there are few particular modifications depending on the constitution of these substances.

The gases have much less density than solids or liquids, and their refraction is much less at the same angle. To render it sensible, therefore, we are obliged to increase considerably the refracting angle of the prism in which they are confined. Borda had one constructed with an angle of 145° 7′ 28″, with a large cylindrical, hollow tube of glass, the two ends of which were cut into a prismatic form, and closed by glasses with parallel faces, carefully luted. The tube was pierced at bottom, and provided with a cock, capable of being attached either to an air-pump or to a receiver, by which means a vacuum might be produced within the prism, and the gases under examination introduced. We have before said, that for the same substance the refraction is changed by a change of density; but the density of gases varies rapidly with a change of temperature or pressure. To be able to compare the results of different experiments, we must take into account these two elements.

40. To measure the pressure, we attach to the prism a verti-
Fig. 34. cal tube *TV*, communicating with its interior, and enclosing a syphon barometer the open branch of which is long enough to permit the mercury to rise to a level, when a vacuum has been produced in the prism. The height at which the mercury of this barometer is supported by the gas within, determines the pressure. To ascertain the temperature, we might insert into the prism a small thermometer; but it would be necessary to place it in the middle of its capacity, which would intercept the light; on this account it is better to suspend two very sensible thermometers without the prism, and very near it, or even in

contact with its faces. The temperature of these faces, as indicated by the thermometers, may be taken without sensible error for that of the gas and air which touch them within and without; for we know how very easily gases acquire the temperature of surrounding objects. We take every precaution, moreover, that the temperature of the place where our experiments are performed, may vary but little during the experiment, and especially that it may vary very slowly.

This prism is then mounted upon a foot perpendicular to its length, by which it is fixed in a horizontal position. The place of observation and the object which we look at should be chosen in such a manner that this object may lie in the horizontal plane passing through the centre of the prism. We then observe the deviation with a repeating circle, the limb of which is also placed in the same plane, at first by approximation, and afterwards exactly, by the condition that the upper telescope, being turned from the direct object to the refracted image, they shall both be found to be on the same horizontal wire, stretched in the interior of the tube. To verify this horizontal position of the wire, it is well that the signal be placed in one of the faces of some large building which presents in its construction long level lines, by one of which we may be governed. Then the best of all sights is a vertical lightening rod which throws a dark line upon the vault of the heavens.

41. Here as well as in the case of solids and liquids, the method of observation consists always in directing the upper telescope of the circle alternately toward the direct object and the refracted image, in order to measure the angle of deviation. But as the deviation for gaseous substances is always extremely small, even with the large prism here supposed, it is necessary, in order to obtain its value exactly, to multiply our observations, and to take the mean of the results, that opposite errors may balance each other. This is done by a repeating process, founded principally on turning the prism from right to left, and from left to right alternately, so as to admit of our observing the deviation successively with the same telescope in these two positions, as is represented in figure 35.

We naturally attempt first to measure the refraction of atmospheric air. In this case we exhaust the air from the prism by means of the air-pump. This operation does not produce an

absolute vacuum; but when the density of the interior air is very much reduced, so as to support the barometrical column at the height of only a small part of an inch, this height is observed, and account taken of it in our calculations. We have, therefore, a prism void, or nearly void, of air, immersed in the surrounding atmosphere; the luminous rays must consequently, in penetrating it, suffer a deviation, determined by the excess of the refracting power of the exterior air, and this in fact takes place. If the upper telescope of the circle is first directed immediately to the object through the air, when we afterwards come to interpose the prism, the deviation is considerable; this is the effect of the refraction of the air. If the telescope be brought again to the object, by moving the limb, and the prism be then turned half round, the deviation is doubled, and the object twice as much displaced. For example, in our experiments, the prism was placed in one of the chambers of the Luxembourg facing the observatory, whose lightening rods were the points of sight. The turning of the prism carried the wire of the telescope from one side of the building to the other; or, to speak more exactly, the telescope remaining immoveable, the edifice seemed to move to the right and to the left of the wire the whole of this distance. Yet we could perceive no sensible dispersion, though undoubtedly there was one produced; but it was too small to be perceptible.

42. If we would observe the refraction of the air at different densities, the process is the same; we only exhaust the air to the proposed limit, which is indicated by the interior barometer.

When we wish to observe other gases, we must first exhaust the air from the prism as far as possible; observe the density of what remains, and then introduce the gas. This introduction is effected by means of a pneumato-chemical bath of water, or of mercury, if the gas is liable to be dissolved in water. It is necessary that the prism, and the vessel containing the gas, should be connected by a double stop-cock, as in the weighing of gases, in order to avoid the water-bubbles which might make their way into the neck of the instrument.

If we wished to obtain a dry vacuum in the prism, or dry gases, we should place in the glass tube which surmounts it a quantity of caustic potash to absorb the humidity. When this and similar substances act in a void, the absorption is almost

instantaneous; but in the air or in gases, a certain time is necessary for the vapours to be precipitated and to combine with the alkali. If, on the contrary, we wished to observe the refraction of aqueous vapours, it would be necessary to employ every means to moisten the air in the place where we make our observations, by sprinkling water, suspending wet cloths, and especially by raising the temperature; but we must avoid introducing these vapours into the prism, for being deposited upon its faces they would affect the passage of the light.

In all that we have said, we have supposed that the glasses which form the faces of the prism have their two surfaces exactly parallel. When great care is used in their construction this is perhaps nearly true, but it is highly improbable that the condition is ever rigorously fulfilled. But as the refraction of glass is very powerful, while that of the air is feeble, it is easy to see that an error of this kind must very much affect our results. To determine its effect, we open the stop-cock of the prism or even detach the glass tube which surmounts it, in order to give free access to the external air. We then observe the deviation in these circumstances, as we should with the prism void or filled with gas. If the surfaces of the glasses are exactly parallel, the object will not be removed from its place by turning the prism, since the interior and exterior air of the prism will be exactly homogeneous and of equal density; but if we observe any deviation, it will necessarily be produced by a defect of parallelism; and this quantity must be added, with its proper sign, to each of our other observations; for it is with this as with all very small quantities, of which the partial effects are only to be added to each other to obtain the total effect.

Having now explained every thing which concerns the arrangement of the apparatus and the manner of making the observations, it remains only to determine the ratios of refraction of the air and of gases. This is a simple subject of calculation.†

† For the necessary formulas see Biot's *Traité de Physique*.

Of Spherical Lenses.

43. The methods which we have employed to calculate the deviations of the luminous rays in their passage through prisms terminated by plane faces, may be applied to the general case in which the refracting medium is terminated by any curved surfaces whatever. For here, as in reflection, we may compare the luminous rays to mathematical straight lines, whose refraction for each point of the surface takes place in exactly the same way as it would do in the tangent plane. It is sufficient, therefore, to calculate the positions of this plane for each point of incidence, in order to determine the deviation of the luminous ray; and this calculation is always possible when the form of the surface is given.

In the common applications of optics it is not necessary to make our calculations so general; for in our experiments we make use of spherical glasses only, since there are no other forms which can be exactly and easily executed; it is sufficient, therefore, to analyse and calculate the refractions which these produce. To do this with all possible simplicity, and to comprehend the results indicated by analysis, we must first take a general view of the sort of glasses in question and make ourselves acquainted with their principal properties.

If we imagine a straight line, or *axis*, drawn through the centres of the two spherical surfaces which terminate such a glass, and that a cutting plane then passes through this axis, we shall have the *profile* of this glass, which, according to the directions of the curvatures that may be given to the two faces, will necessarily have one of the forms represented in figures 36, 37, 38, 39, 40, 41. These different forms are distinguished by the following names which are generally adopted.

Fig. 36. (1.) The double-convex glass. This glass is called a *lens*, (the Latin word for a lentil, which it resembles in shape,) and the name has been extended to all the other spherical glasses.

Fig. 37. (2.) The plano-convex lens. We speak of the concavity or convexity, always, in relation to objects situated without the glass.

(3.) The concavo-convex. The difference of these two forms is, that the first is thinner at the edge than at the centre; and the second, on the contrary, is thicker at the edge than at the centre. We shall soon see the peculiarities which result from this difference of construction. Fig. 38, 39.

(4.) The plano-concave. Fig. 40.

(5.) The double-concave. Fig. 41.

All these forms of lenses agree in this, that the planes tangent to the two spherical surfaces which terminate them, are at first parallel to each other at the points A_1, A_2, where the lens is pierced by its axis; from this point to the edge of the glass, the angle of the two tangent planes goes on increasing more and more, and symmetrically on each side of the axis. A luminous ray, which passes through such a glass, is refracted precisely as it would be in passing through a prism formed by the two planes tangent to the points of incidence and emergence. A spherical lens of any form whatever, may be considered therefore as an assemblage of such prisms, or as a prism of a variable aperture, the refracting angle of which, being nothing at the axis A_1A_2 of the lens, afterwards goes on increasing to its edges.

Accordingly all the forms of spherical glasses which we have been describing, may be divided into two classes, according as the base or the point of the refracting prism is turned toward the axis A_1A_2, of the lens. The first class will comprehend the figures 36, 37, 38; the second the figures 39, 40, 41.

It is easy to understand the influence of this different arrangement of the prisms upon the course of the luminous rays. For if we suppose a beam of incident rays parallel to each other and to the axis A_1A_2 of the lenses, it is evident that all those of the first class will refract these rays toward the axis A_1, A_2, while those of the second class, on the contrary, will turn them from it. Thus the first will cause the incident light to converge, and the latter will make it diverge; these two kinds of lenses are hence called *converging* and *diverging glasses*.

44. Let us examine more particularly the manner in which these phenomena are produced, and let us begin with the first kind of lens, of which we have a general representation in figure 42. Among the rays which compose the incident beam parallel to the axis A_1A_2, there is one SA which coincides with this axis itself. This passes through the lens at the points where

the two surfaces which terminate it are parallel. Moreover, its incidence and its emergence take place perpendicularly to these two surfaces. It therefore suffers no deviation, but passes on in its original direction SA_1A_2F. But it is not the same with the incident rays situated at a little distance from the axis. These suffer a very slight refraction on account of the small refracting angle of the prism through which they pass. They will, therefore, cut the first ray somewhere as in F. As the incident rays depart from the axis, their deviation increases, and they will cut each other successively in $F_1, F_2, \ldots\ldots$, and the assemblage of all these intersections, supposed, when taken two and two, to be infinitely near to each other, forms in general two branches of a curve which begin at the point F, where the rays nearest the axis cut each other, and terminate at the point F in the last ray which passes through the edge of the lens. These curves are named *caustics*. But when the surfaces of the lens include but a few degrees of the spheres upon which they were formed, we find by experiment that the greater part of the rays meet at F, rather than in any other place, so that the curve FF_1F_2 is almost entirely concentrated in that, to which we give the name of the *principal focus*. The distance of this focus is sensibly the same for every lens, whatever be the distance of the faces which are presented to the incident rays.

45. Reasoning in the same way with respect to diverging
Fig. 43. lenses, of which the general form is given in figure 43, it will be seen that they must form two branches of a curve FF_1F_2, also symmetrical above and below the axis; but the *principal focus* F of the rays near the axis falls on the same side of the lens with the incident rays; and there is no real concentration of light at this point, nor at any other point of the curve of intersection. This curve then indicates only the imaginary place of meeting of the emergent rays produced.

In all the figures which we have yet examined, the lenses are represented as perfectly symmetrical about the axis A_1A_2, so that this axis contains also the centre of figure of their exterior surface. In this case, the glass is said to be *exactly centred;* and this is a very important condition in all optical experiments, as we immediately perceive. When it is not fulfilled, the thickness of the lens at its edge is necessarily unequal, as appears in
Fig. 44. figure 44, in which A_1A_2 is really the common axis of the two

spherical surfaces while B_1, B_2 is the apparent axis drawn through the centres of the two circles which form the exterior contour of the glass.

Hence it follows that converging lenses are necessarily centred when they are sharp at the edges; for their thickness at these edges being nothing, it will evidently increase uniformly on every side toward the centre of the figure, where the two surfaces will be parallel. When we have learned how to determine by experiment the position of the foci, it will be seen that we may make use of this determination with much accuracy to verify the centring in every kind of lens.

46. After what we have said above concerning the formation of caustics, it will be readily perceived, that in lenses as well as in mirrors, the concentration of the rays will be so much the more perfect as they pass nearer to the axis of the lens. Hence in optical instruments, we are often obliged to cover the edges of the lenses and a portion of their surfaces with opaque circular rings, which are called *diaphragms*. The luminous rays fall then only on the circular and central portion of the lens which has not been covered. The diameter of this remaining portion is called *the aperture of the glass*.

In optical experiments which require much accuracy, this aperture is made very small compared with the radii of curvature of the lens, and no rays are admitted but such as are very little inclined to the axis which joins the centres of its surfaces; these are the only means of obtaining distinct and well defined images. It hence results, that both in their incidence and their emergence, the luminous rays meet the surface of the lens almost perpendicularly; which reduces the deviations they experience, and greatly facilitates the calculations by which we determine them.

47. To fix the circumstances of the passage of the rays and the formation of the image geometrically, let us first consider a single radiant point S, placed before the first surface of a spherical lens. Through this point and the axis of the lens, draw a plane cutting the glass in the direction of one of its profiles A_1A_2MM. It is always to be understood that the rays proceeding from the point S are, during their whole course, very little inclined to the axis A_1A_2X, and that their points of incidence and emergence I_1, I_2 are very near this axis in com- Fig. 45.

parison with the radii of the two spheres to which the surfaces of the glass correspond.

48. If several spherical lenses are placed on the same axis with the first, and the beam of light from the point S passes successively through them all, it is evident that such of the rays as are comprehended in the plane represented in the figure will continue in that plane, since it is perpendicular to the surfaces through which they pass. But the rays which vary from this plane either above or below, will pass successively into different planes of incidence and refraction, which, it would seem, must render their course very difficult to be calculated. Fortunately, this calculation is unnecessary when the angles of incidence and emergence are small, as we must always suppose them in optical instruments; for these rays are collected into very nearly the same foci with the others, so that it is sufficient to follow out the first rays in order to determine the place where the image of each radiant point is formed. Hence we have only to consider the course of the rays comprehended in the plane drawn through the radiant point and the common axis of all the lenses.

Here, as in the case of spherical mirrors, all our results may be deduced from the principal focal distance, and the method is the same. It is this distance, therefore, which we must first determine; and this is easily done when we know the radii of the two surfaces of the lens, and the ratio of refraction which belongs to its substance. The focal distance is equal to the product of these two radii, divided by their difference, and by the ratio of refraction diminished by unity. This supposes the curvatures turned in the same direction; if they are in a contrary direction, we must take the sum of the radii, instead of their difference.

Fig. 38, 39.

Fig. 36, 41.

This being laid down, we may easily find the focus of any radiant point situated either in or without the axis. For, let S be this point and MAA_1M the profile of the lens, which I represent by a straight line to indicate that it is supposed to be very thin. Through the point S draw next the incident ray SI parallel to the axis AA_1X; this ray after the two refractions, will pass into the principal focus F, so that IF will be the direction of the emergent ray thence resulting. Draw now another incident ray SA, directed to the centre of figure of the lens; this will pass through the glass without deviation, since

Fig. 46, 47.

the thickness being supposed infinitely small, and the two surfaces at AA_1 parallel, the lens has at this point the effect of an infinitely thin plane glass. It only remains, therefore, to produce SA in a straight line, till it cuts the first emergent ray in f; the point f will be the common focus of these two rays, and it will be the focus also of all those which proceed from the same radiant point S. Figure 46 represents the effect of this construction for the converging lens, and figure 47 for the diverging lens. Expressing our operations algebraically, we obtain a general formula for determining the length of the focal distance Af, and the position of the focus, for all possible curvatures of the surfaces, and all situations of the radiant point. Hence it is easy to determine the images of objects which have finite dimensions, for we have only to apply the same construction to all the cones of rays proceeding from the different points which compose it. We shall thus find the foci of these several cones, and they will together form the image of the object.*

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* As the formula indicated in the text is one that is very often used, I shall give an explanation of it here, making use of figure 47. We designate by Δ the distance AS of the radiant point from the centre of the lens, supposed to be infinitely thin. We denote also by Δ' the distance of this centre from the focus f, supposed on the same side with S. We call r the ray of the anterior surface of the lens situated on the same side with the radiant point S, and r' the ray of the second surface, both surfaces being considered as having their concavities turned towards S; and lastly, we designate by n the ratio of refraction for the kind of glass of which the lens is made; the distance Δ' will always be connected with Δ by the following relation,

$$\frac{1}{\Delta'} = \frac{1}{\Delta} + (n-1)\left(\frac{1}{r} - \frac{1}{r'}\right).$$

In this formula the rays r, r', are considered as positive when the surfaces to which they belong are concave towards the radiant point. If, on the other hand, one surface is convex towards this point, we must give its radius the negative sign. Also the distances Δ, Δ', are considered positive when they are situated on the side of the radiant point, as in figure 47. Therefore, if one of them, Δ' for instance, becomes negative by the disposition of the values of Δ, r, r', this will signify that the focus f is formed on the opposite side of the lens; consequently beyond it, as in figure 46, and not on this side, as

Determination of the Images produced by Diverging Lenses. Use of these Lenses in correcting shortsightedness.

49. We shall first apply the method to diverging lenses. Let
Fig. 48. MAM be such a lens, of which A is the centre of figure; and place the object SS' before its surface, at any distance whatever, provided it be such that the angles of incidence shall not exceed the limits supposed in our approximations. If from the extremity S of the object, we draw the line SA to the centre of figure of the lens, the cone of incident rays proceeding from the point S, will have for its axis SA, and its focus will be found somewhere in this straight line, and on the same side of the lens (since it is diverging), for example, in f. The focus will be found in the same way in $S'A$ as at f'; and these two foci comprehending between them all the others, ff' will be the image of the object. It will always be erect and smaller than the object, since it is comprehended between the sides of the angle SAS', and nearer its vertex A. Moreover, the absolute value of its distance from the lens will always be less than the principal focal distance AF, and so much less according as the object itself is nearer the glass.

According to this construction, when the luminous rays, proceeding from the same point S or S' of the object, have passed through the lens, their course is exactly the same as if they had set out from the corresponding point f or f' of the image. There-

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in figure 47. This always happens, for instance, in converging lenses, when Δ is infinite, which renders $\frac{1}{\Delta}$ nothing. For then, according to figures 36, 37, 38, the ray r of the anterior surface becomes negative; and whether the other lens be convex or concave, $\frac{1}{r} - \frac{1}{r'}$, will remain a negative quantity. We shall then have

$$\Delta' = -\frac{r r'}{(n-1)(r-r')};$$

this is precisely the rule enunciated in the text for the calculation of the principal focal distance AF.

fore, if a spectator had his eye situated at OO on the other side of the lens so as to receive all these rays or only a part of them, he would see neither the points S, S', nor the points between them, but their image ff'; and his eye would be affected in the same way as if the object had really become smaller, and had been transferred to the place where the foci are formed. He will thus see this imaginary object erect, diminished and brought nearer. But although these may be in fact the only elements of the sensation produced in the eye, yet we do not estimate correctly the distance and magnitude of the image, because our judgment is affected at the same time by other considerations, altogether independent of the direction of the luminous rays.

To convince ourselves of this singular fact, we may take any diverging glass, for example, that nearest the eye in an opera glass, which is usually double concave; and look through it at objects which we will suppose to be very distant compared with the focal distance of the glass. When the eye is placed at a proper distance from the posterior surface, we shall see a very distinct image of these objects. It will appear erect like them and smaller, but instead of supposing it near the glass and in the focus ff', where it is really formed, it will seem to us more distant than the object itself. This is because the sensation of the visual angle, and that of the greater or less divergency of the luminous rays which reach us, are not the only elements from which we estimate distances. We add to these, without being sensible of it, the impressions which we may have of the absolute dimensions of objects. A man seen successively at the distances 20, 40, and 60 yards, appears always of the same absolute magnitude. Nevertheless the rays of light which render him visible at these different distances, cross each other on entering the eye under very different angles, since they are to each other nearly as the numbers 1, $\frac{1}{2}$, $\frac{1}{3}$; so that were we to judge only from the openings of these angles, the apparent magnitudes would seem to us to decrease in the same ratio.

50. This habit of connecting the idea of absolute magnitude with the sensation of the visual angle in judging of the distance of objects, is derived from the experience of our whole life, and it becomes as rapid as sensation itself; or rather, the sensation which is transmitted to the mind when we look at an external object, is the compound result of these different elements. But

the involuntary application which we make of them, deceives us when we look through a diverging lens; for the objects which we had just viewed with the naked eye, of the absolute magnitude and distance of which we were consequently able to form a pretty accurate judgment, being suddenly presented to the eye with much smaller dimensions, we do not hence conclude simply that their images are smaller, but that they are further removed from us; and no reasoning can prevent this conclusion being formed even when we know theoretically that it is false.

With the exception of this case, in which our senses are but poor judges, observation perfectly confirms the results indicated by theory; this we may verify not only when the object is at a great distance, as we have just now supposed, but also when it is gradually brought near. If it be brought, however, very near the lens, it must be very small, and the refracting surfaces must be held nearly perpendicular to the rays which proceed from it; otherwise we should exceed the limits of incidence and emergence supposed in our approximations.

In performing these experiments, we find that in order to see the image distinctly, we must place the eye at a certain distance from the lens, and this distance varies with different eyes. If the eye be brought nearer, the image becomes larger and confused. If, on the contrary, it be removed, the image becomes smaller and more difficult to be seen distinctly. This results from the fact, that the eye is itself an optical instrument which cannot concentrate the rays with sufficient exactness for perfect vision, except when they fall upon its surface within certain limits of incidence.

Suppose, for example, that the luminous point S, forms its
Fig. 49. image in F, and that this image appears distinct when the eye is placed at OO; then vision is produced by a cone of rays FOO which has for its base the surface OO of the pupil, and for its vertex the point F. If the eye be brought nearer, suppose to $O'O'$, then the pupil intercepts a more diverging cone of rays, and consequently the rays FO' which form the exterior surface of this cone, fall upon the eye with a greater angle of incidence. If this angle become so great that the eye cannot concentrate all the rays upon its retina, vision is necessarily confused; and this in fact takes place when we bring the eye too near the lens, and consequently too near the focus F, the common centre of the

emerging rays. If, on the contrary, after having found the point where vision is most perfect, we remove the eye further from the lens, the image, which remains always in the same place, is at a greater distance from the eye. It ought, therefore, to appear smaller and less distinct in its outline, like other objects when removed from us; and this actually happens.

51. The least distance at which objects are seen differs with different eyes. Those persons who are *shortsighted* are obliged to approach the focus F; those who are *longsighted*, on the contrary, find it necessary to go to a greater distance from it. The reason of this will be readily understood when we remember that this focus, from which the rays diverge, is to the eye as it were an object situated in F; and thus each one, in order to see it distinctly, is obliged to place his eye at the distance at which he would see a common object most distinctly. This distance is commonly from eight to ten inches, when we wish to distinguish the minute parts of small objects; but some persons cannot distinguish objects placed at so small a distance; and, on the contrary, there are others with respect to whom this distance is only two or three inches. The latter are called *myopes*, or *shortsighted* persons, and the former *presbytes* or *longsighted*.

52. It is not to be understood that distinct vision is, in every case, strictly confined to a particular distance. On the contrary, the eye is endued with a power by which it is enabled to adapt itself, within certain limits, to the different distances of objects. But beyond these the image is confused, and vision becomes imperfect. Thus persons possessing the longest sight, cease to distinguish the minute parts of objects when placed at a considerable distance. But these parts disappear to shortsighted persons, even when the objects are very much nearer.

The defect of shortsightedness may be corrected by placing a diverging lens between the object and the eye, as represented in figure 49. For by means of such a lens, there is substituted instead of the real object the image formed at its focus, and we have only to give to the lens a focal distance equal to that of distinct vision of the organ for which it is designed; then by bringing it near the eye, the shortsighted person will see distant objects as distinctly as if they were situated near him, although he refers them to their true places. But he cannot use the same glasses, at least, when brought close to the eye, for the purpose

of seeing very near objects; because the foci of the rays which proceed from them being formed still nearer the glass, the image of the objects would be too near the eye to be seen distinctly without fatiguing it. It is necessary, therefore, to employ lenses of a greater focal distance; or, which is better, to dispense with them entirely, since it is these near objects which shortsighted persons perceive most distinctly with the naked eye, and to reserve the use of diverging glasses for distant objects. The principal focal distance ought, moreover, to exceed a little that at which small objects are seen most distinctly; for the eye would be soon fatigued by the too near approach of the image. Such is the purpose and operation of those *spectacles* by which shortsighted persons remedy the imperfection of their sight.

It is plain that the glasses in question would be useless and worse than useless to the longsighted who cannot perceive near objects distinctly, on account of their being too close to the eye; for, the foci of diverging lenses being always nearer than the object itself, the evil would only be increased. We must, in this case, therefore, seek some means of removing the image beyond the object which produces it; and this purpose is effected by means of converging glasses, as we shall soon see when we have studied their properties.

Method of determining the Images produced by Converging Lenses. Use of these Lenses in correcting the defective Vision of longsighted Persons.

Fig. 50. 53. Let MAM be a lens which we now suppose to be converging; and let SS' be the object placed beyond the focal distance of parallel rays. Through its upper extremity S draw the ray SA to the centre of the lens. This ray will be the axis of the pencil of rays proceeding from the point S; and the lens, being supposed to be very thin, the focus of this pencil will be situated somewhere beyond the lens with respect to the point S, as at f, for example, in SA produced, as appears from our general construction. All the emerging rays which come from the point S will meet in this point from which they will pass on and diverge as if from a real object situated in the same place. If

we repeat the same construction for the other extremity S', we shall find its focus f' in the axis $S'A$ produced; and in the same way we obtain all the foci of the intermediate points; whence we have ff' an inverted image of the object. The image is inverted because the foci are formed beyond the point where the axes of the pencils cross each other.

This image will become sensible if it is received upon a screen of white pasteboard or ground glass placed in ff'; it may also be seen immediately, if the eye be situated beyond this point, at the proper distance for seeing distinctly a real object occupying the same place.

54. If the luminous object SS', be situated at a very great distance, the image will fall on the opposite side of the lens, near the principal focus F. This furnishes us with the means of determining experimentally the focal distance of converging glasses. As the object approaches the lens, the image recedes from it increasing at the same time in magnitude. When the object is removed only by a quantity double the principal focal distance, Fig. 51.
the image will be of the same magnitude; if it be brought nearer, the image will be removed, the dimensions being increased; finally, when it arrives at the principal focus the image will be Fig. 52.
removed to an infinite distance. This result might be easily foreseen, since the object and image may always be made to change places; if the object, placed at an infinite distance, has its image in the principal focus; reciprocally, when the object is placed in the focus, the image will be at an infinite distance. Between these two limits, the image is always inverted.

The object, being brought nearer continually, will fall at length between the principal focus and the surface of the lens. Then Fig. 53.
the image, according to our general construction, passes to the same side of the lens. It is now larger than the object, more distant and erect; as the object approaches the lens, the image approaches also, the size being diminished; till finally the image and object unite and coincide throughout upon the surface.

55. These are the results of theory, and they are completely confirmed by experiment. Take, for example, any converging lens, as the object lens of an opera-glass, and look through it at an object which we will at first suppose at a great distance relative to the focal distance of the lens. Then, plac- Fig. 50.
ing the eye properly we shall see an inverted image of this

object, which, as has already been remarked, we can render sensible by receiving it upon a screen of pasteboard or ground glass. But although this image really falls on the same side of the lens with the observer, yet it is referred by him to the opposite side, and conceived to be at a greater or less distance than the object, according to the circumstances by which his judgment is influenced in other cases.

56. When we are thus placed at the point where the image is most distinctly seen, if we measure the distance of the eye from the lens, we shall find that it is equal to the focal distance of parallel rays, plus the ordinary distance of distinct vision for the eye of the observer. This is another proof that the eye is placed so as to view the image as if it were a real object; and it receives from the image similar impressions. This opinion is every way confirmed; for if the eye be removed further from the glass, the image appears diminished and its minute parts are less easily distinguished, like those of a real object which is removed from us. If, on the contrary, we approach the image, it becomes irregular and confused, like that of an object which is brought too near the eye. In this last case, the image appears to be increased in size, like an object seen very near; and we are led to believe that it is brought nearer. It finally becomes altogether confused and indistinct when the eye arrives at the focus itself. But, which is very remarkable, as we approach still nearer the glass, the image is formed again, erect and very much confused. Its direction is not again changed as the distance of the eye from the glass diminishes, but it becomes less indistinct, and at length we see the object tolerably well, with its natural outlines and dimensions, when the eye is at the surface of the glass; especially if we contract the aperture of the pupil by looking through a small hole made in a card.

In these last experiments the rays are convergent when they approach the eye; and since the image is still perceived, it is a proof that vision may also take place in this way, though with incomparably less distinctness than when it is produced by divergent rays.

57. But it may here be asked why the image appears erect and becomes less confused as the eye approaches nearer to the glass. To resolve these different questions clearly, let us first take the simple case where the radiating object is reduced to a

luminous point very far removed. This takes place, for example, when we look through a converging lens at Venus, or any very brilliant star.

In this case, if the eye is first placed in OO, beyond the focus of parallel rays, and at a proper distance for distinct vision, we shall see a very clear well defined image of the star. This sensation is produced by a cone of divergent rays which has its vertex at F, and OO, the aperture of the pupil, for its base. As the eye approaches the lens, the base of this cone remaining always of the same magnitude, it intercepts a greater number of rays, and of rays which form among themselves a greater angle. On account of this divergency the eye is unable to collect them all into the same focus on the retina; and consequently, they form upon this membrane a small circular image, such as a luminous circle would produce placed without the eye. The image of the star is thus gradually enlarged, and forms a disc which increases in size as the eye approaches the focus. Finally, when the eye arrives at $O'O'$, that is, at the focus, this disc is equal in magnitude to the lens itself, because the aperture of the eye then admits all the rays which the lens has refracted. But as the eye approaches still nearer the glass a certain portion of the rays are lost; and those which first escape, being those which depart farthest from the axis of the lens, they are also the most convergent. Hence, when the eye is very near the glass, the only rays which can enter the pupil have very little convergence, and consequently the object begins to appear less confused. This being applied successively to all the points of an extended object, we perceive why it appears more distinct as the eye approaches the glass. The position is erect, since the luminous pencils do not cross each other before they enter the eye. Fig. 54.

58. Having thus verified the results of theory for very distant objects, we shall now consider those which are placed at a less distance. In this case we also verify the other phenomena indicated by theory, and we find them the same, whichever of the faces of the lens is presented to the incident rays.

It will hence be readily understood what advantage is derived from the use of converging lenses by long-sighted persons who see near objects confusedly. For, if the object SS' be too near the eye OO to allow the rays, proceeding from it, to come to a focus upon the retina, we have only to interpose the convex lens Fig. 55.

MAM, so near the object as to bring it within the principal focal distance *AF* of parallel rays, and having such a convexity as to throw back the images of the points *S*, *S'*, to the foci *f*, *f'*, precisely at the distance *f O*, *f' O*, where the eye sees most distinctly. This is the way in which convex glasses enable long-sighted persons to see near objects distinctly. They are particularly important to aged persons, who are commonly long-sighted, and who are able by means of this useful invention to read, write, and in general to execute any kind of work which requires to be placed near the eye, as if the focus of vision was not affected by age. But they are obliged to dispense with these glasses in looking at distant objects, because they make the rays converge too fast, and thus occasion indistinctness of vision, as we have already shown.

Of Magnifying Glasses and Simple Microscopes.

59. THE expedient above mentioned is useful also to watchmakers, engravers, and generally to those persons whose business requires much care directed to small objects. In this case, however, it is not to remedy defects of sight, but to magnify the objects,
Fig. 56. and render the minute parts more perceptible. To understand this use of lenses, let *OO* be the eye, and *SS'* the object which we wish to see distinctly and in large dimensions. If the latter condition were the only one sought, it might be fulfilled by simply bringing the object nearer the eye, for the visual angle subtended increases as the distance diminishes. But the image would appear confused since the object is brought within the limits at which distinct vision naturally takes place. To remedy this inconvenience, we have only to place close to the eye a convex lens, and to bring the object within the focal distance of the glass precisely so far that its image shall be thrown back to the distance *Of*, *Of'*, of distinct vision. This is always possible; for the distance of the image from the glass may vary from nothing to infinity, according to the distance of the object. When we have found by several trials this proper distance, the image *ff'* will be seen distinctly; and, moreover, the visual angle *fAf'*,

which it subtends at the eye placed in contact with the glass, is equal to the angle *SAS'* under which the object would appear to the naked eye, if it could be seen distinctly at so small a distance as *AP*. We shall thus obtain the double advantage of seeing the object distinctly, and of seeing it under a larger angle. But from the very circumstance of never having seen it in this way, the judgment which we form of its real magnitude is not modified by any previous experience of the ratio of the distance to the visual angle; and as we see it under a much greater angle than it subtends to the naked eye, although at the same distance at which we endeavour to place it in order to see it distinctly, it must appear to be magnified in all its dimensions. And this is constantly found to be the case. Convex lenses fitted to produce the effect in question, are called *magnifiers*.

60. The magnifying power is evidently determined by the ratio of the absolute magnitudes of the image and object; and this ratio is the same, by the construction of the figures, as that of their distance from the glass, since they are both comprehended between the sides of the same angle of fAf'. Thus the enlargement takes place because the image is formed at a greater distance from the glass than the object, and on the same side of it. These two conditions cannot be fulfilled, except with converging lenses.

61. We have supposed the eye to be very near the surface of the glass. This position renders our considerations more simple; and affords a wider field, that is, it allows us to take in with the same glass, a larger extent of objects. But after what we have said, it is evident that this condition is not indispensable to the obtaining of magnified images. Indeed, whenever the image, although larger and more distant than the object, is too near the glass for distinct vision, by removing the eye we can always find the exact point where it may be clearly and distinctly seen.

Large magnifying glasses are sometimes employed in this way by aged persons placed at some distance from the eyes, which enable them to read. They are also used in examining the minute parts of maps. It is obvious that these glasses must be very large that both eyes may look through them at once. It is necessary, moreover, that the focal distance should be so great that the axes of the luminous pencils proceeding from the same

object toward the two eyes, may not make too great an angle with the common axis of the two surfaces. The magnifying power produced by these glasses, necessarily varies with the distance at which they are held from the object to be examined, as appears from the preceding considerations; and their effect is the more perfect according as they are held near the object, since on the supposition that we look through the centre of the glass, the luminous pencils which enter the pupil of each eye, pass through the glass under small angles of incidence, agreeably to what has been taken for granted in all our approximations. But the condition of seeing with two eyes obliges us to incline the pencils to the axis of the lens more than if we used but one; and this circumstance, together with the facility with which we alter the magnifying power by varying the distance of the object from the lens and of the lens from the eye, makes this instrument liable to injure the sight. It is, therefore, at present rarely used, and we employ instead of it separate glasses for the two eyes.

62. We generally use but one eye with magnifiers and other optical instruments in which we wish to obtain a highly magnified image with the greatest possible distinctness. In this case experiment proves that the best position of the eye is very near the glass. It only remains, therefore, to place the object within the principal focus, at such a distance as to throw the image back to the point of distinct vision; and this is effected after a few trials. The calculus also furnishes rules for determining this position with respect to every lens, and the distance of distinct vision belonging to different eyes, while it teaches us also how to calculate the magnifying power thus obtained. To form a glass which shall magnify a certain number of times m, when applied to the eye, we must give it a focal distance equal to the distance of distinct vision divided by the magnifying power minus unity, that is, by $m - 1$. For example, if we take for the ordinary distance of distinct vision eight inches, which is the usual measure for good eyes, and if a glass is sought which shall magnify fifty times, its focal distance must be $\frac{8}{50 - 1} = 0{,}163$; if it were required to magnify 100 times, the focal distance must be $\frac{8}{100 - 1} = 0{,}0808$; which, supposing the two curvatures the

same, would require each surface to be of a less radius than the above. In general, the radii of the surfaces diminish as the magnifying power increases. This is an obstacle to carrying the magnifying power very far with a simple lens. It will be seen also that the focal distance would diminish if the distance of distinct vision were less than eight inches; and it is for this reason that the same glass magnifies less with respect to short-sighted persons, than when used by those who are long-sighted.

63. We thus see how far we can proceed with a single microscopic lens. But by combining these lenses with others of less curvature, we obtain much more powerful effects. By similar combinations of large glasses, or of mirrors and magnifiers, we are able to form arrangements which bring near and magnify the images of distant objects; and thus we obtain what are called *refracting and reflecting telescopes.* We shall make known the theory of these instruments when we have learned how to correct the effect of the decomposition of light in the case of simple lenses, which cause the images to be coloured, and is on this account the occasion of an essential defect.

64. A very simple means of procuring a microscope of considerable power is by making a small hole in a thin metallic plate, and introducing into it a drop of water. This drop arranging itself in the hole, as in a very narrow and very short capillary cylinder, forms at the two surfaces of the plate two spherical convexities, whose diameter is the same as that of the hole. Then the rays which pass through this globule are refracted by it as by a solid glass magnifier of a very short focus. Then placing the eye very near the hole and looking through it at small objects, situated on the other side of the plate, at a very small distance, we find a point where they may be seen distinctly and very much magnified.

65. The lenses obtained by this process are inconvenient, as they are continually changing their curvature, and are soon dissipated by evaporation; this renders them unfit for continued observations. Dr Brewster proposed to remedy the defect by using drops of transparent varnish on a plate of glass, which, by the mutual attraction of the particles, take a spherical shape. Very clear lenses are obtained in this way; but it is difficult to fix them in a metallic plate, and still more difficult to keep them unaltered. By a slight modification Mr Sivright of Edinburgh

has discovered a process that is free from all these inconveniences. It consists in placing small fragments of glass on perforations in a thin plate of platina, and then submitting them to the action of the blow-pipe. The glass thus melted, assumes a spherical shape like the drop of water or varnish; when this is once cold, it preserves its form, and presents a lens well mounted. Instead of employing a plate of platina, we can make use of a wire of the same metal rolled around a fragment of the glass, which serves the purpose of a ring to keep it in place. Platina is preferable to any other metal for this purpose, because it does not oxydate with the heat of the blowpipe, and it adheres very strongly to the sides of the small lens; its point of fusion also is much higher than that of glass, and it may be employed very thin, so that its contact shall less retard the fusion of the vitreous globule.

In these different applications it is supposed that the lenses used are capable of concentrating the parallel rays which fall upon them in a single focus. But this condition can only be fulfilled when the thickness and aperture of the lens are very small compared with the radius of curvature, and it is also, as we have seen above, the only case which our theoretical approximations embrace. It is manifest that in practice we must conform to these conditions in order to obtain distinct images; and experiment soon shows within what limits we are to confine ourselves. We may nevertheless, by a happy choice of curvatures, increase the extent of our glasses, and at the same time, diminish the aberrations of the focus. We may even destroy almost entirely these aberrations by combining several lenses; but the proper arrangement for producing this effect cannot be discovered without the aid of the calculus. It is sufficient in this place to observe that our arrangements tend generally to diminish the angles of incidence and emergence of the rays, at the different surfaces which they are successively to pass through.

Physical Theory of Refraction.

66. The methods above described give the ratio of refraction for any substance in the state in which it is observed. But it has already been remarked that this ratio, in any case, changes with a change of density. The variation is small in solids and liquids, since the dilatations and condensations to which these are exposed are inconsiderable; but it is quite different with respect to aeriform substances, in which every change of pressure and temperature is attended with a marked change of density. While we are ignorant of the relation which exists between the density of a substance and the refracting power it exerts upon light, we cannot compare together the observations made upon the same substance in different states, nor those made upon different substances; nor will it be possible to distinguish what belongs to their densities, from that which belongs to the chemical nature and arrangement of their particles. In order, therefore, to a more thorough acquaintance with these phenomena, it is necessary, by means of the observed motions of a ray of light, to discover the forces which act upon it, and thence to ascertain by calculation how these forces produce the particular results.

Let *AB* be the first surface of a body or of any refracting medium, and let *SI* be a luminous ray traversing a void in a direction nearly parallel to the surface of the medium, but meeting it at the point *I*. Then this ray will no longer continue its direction in a straight line, but will be refracted in a certain direction *IR*, which will depend upon the ratio of refraction for the medium under consideration. If this were water, for example, the angle *BIR*, formed by the refracted ray and the refracting surface, would measure $41^\circ\ 18'\ 36''$, and the incident ray would be turned thus far out of its course. Now, as a moving body can be made to change its course only by a force oblique to its own direction, we must conclude that the particles of light, in approaching the surface of the medium, are acted upon by forces which tend to make them enter it; and moreover, that these forces are directed perpendicularly to the surface; for the Fig. 57.

case of perpendicular incidence is the only one in which they do not change the direction of the ray.

It is evident that these forces must be sensible only at very small distances above and below the surface. For the luminous ray must begin to bend the moment it begins to be acted upon, and must take a rectilineal direction the moment they cease. Now the space occupied by this inflection is so small as to be incapable of being appreciated by our senses, and the ray appears to be broken suddenly at the point of incidence. The action of the refracting forces is not then sensible to a greater extent.

67. All these results concur to prove that the refraction of the luminous rays is produced by the affinity of the particles of the body for the particles of light; an affinity analogous to capillary attraction, and which, like that, becomes sensible only at very small distances. This conclusion, drawn from the phenomena above stated, appears at first view, to contradict that which we have deduced from experiments upon reflection. For, in that case, the luminous particles appeared to be repelled by the reflecting body, instead of being attracted as we now find them to be. But it must be remarked that the particles which are reflected may not be in the same physical state or in the same circumstances of motion, as those which are refracted. Now the possibility of this difference is sufficient to remove the apparent contradiction in the two opposite consequences, drawn from the phenomena exhibited in these two states of the particles. For when a body *A* acts upon a body *B*, in any manner whatever, this action does not depend simply upon the state and nature of *A*, but also upon those of *B*. We shall hereafter see this difference in the state of the luminous particles confirmed by abundant proofs, and we shall learn also in what it consists.

Let us now determine from these phenomena the conditions to which the attractive forces are subjected. For this purpose, let
Fig. 58. us suppose a luminous particle *M*, placed at any distance without a homogeneous refracting medium, and so modified as to escape the repulsive action of the reflecting forces. Then this particle will be sensible only to the attraction of the medium, which draws it perpendicularly to the surface *AB*, as before demonstrated. Moreover, it will be drawn with the same intensity, at the same distance, whatever part of the surface it approaches, whether it be at *M*, or at *m*, or at μ. For, as this kind of action is sensible

only at very small distances, provided the points M, m, μ, are not situated infinitely near the extremities A and B, of the medium, the luminous particles, supposed to be in these points, will be attracted with as great a force as if the medium were indefinately extended; and the intensities of these attractions must be equal throughout, since we suppose the medium to be homogeneous. Also, in order that we may have to consider, in the action of the medium, only the progressive variations depending upon the distance, let us suppose that it is not crystallized; in which case no account need be taken of the modifications of the attractive force, which might result from the figure of the particles, or their arrangement. Then supposing the distance $a\,A$ to be that at which the attraction of the medium begins to affect the luminous particles, the line $a\,b$, parallel to AB, will represent the exterior limit, at which the luminous ray begins to be bent. It is manifestly not necessary to regard this limit as rigorously and mathematically exact; for, mathematically speaking, the attractive action must extend indefinitely; but since, beyond a very small extent, it becomes so exceedingly feeble that its effect is insensible, we may express this circumstance graphically, by indicating near the surface a certain very small distance, as the limit at which the ray begins sensibly to bend.

Let us suppose now that the luminous particle M, having passed this limit approaches nearer the medium. It will then be more forcibly attracted, and the attraction will increase until the luminous particle reaches the surface of the body. But when the particle has traversed this surface, the attraction of the medium will begin to diminish, and will diminish progressively according as the particle penetrates into the interior of the medium. Suppose it to have arrived at a certain depth M', for example. In order to ascertain the relative intensity of the force which then attracts it, draw through the point M', the line $c'\,d'$ parallel to the surface AB; and below this line draw another parallel $A'B'$, at the same distance from M' with AB. Then the two equal portions of the medium, which are limited on one side by the line $c'\,d'$, and on the other by AB, $A'B'$, will attract the particle M' in contrary directions, and will attract it equally. These two parts, therefore, will counterbalance each other; so that the particle will be attracted only by that portion of the

medium situated beyond $A'B'$. If the thickness of this remaining part exceeds the limit at which the attractive forces are sensible, the intensity of the attraction will be the same as when the particle was without the medium, and as far distant from it as it has now penetrated. For, by reason of the small distance at which the attraction is sensible, the portion of the medium which is cut off by the plane $A'B'$, is always infinitely thin; and the rest of the medium, since it exceeds the sphere of activity of the forces, may be considered as indefinite.

It results from this reasoning, that the limits of the interior and exterior attractions are equally distant from the surface. When the luminous particle has attained this depth, if we cut off from the medium the portions which counterbalance each other above and below, the rest of the medium is too far distant to produce any sensible attraction.

Resuming the results at which we have arrived, we perceive that the incident ray SI preserves its rectilineal direction till it arrives at the first limit $a\,b$, where the attractive forces begin to be sensible. From this point the action of the forces begins to bend the ray into a curve, to which its primitive direction is a tangent at I, and whose concavity is turned towards the interior of the medium, as shown in the figure. This curve continues to the interior limit $a'\,b'$, where the influence of the attractive forces again becomes insensible. Then the ray takes the rectilineal direction, derived from the refraction, and which is the prolongation of the last tangent to the curve it has described. The extent of this curve is too small to be appreciated by our senses, and it is necessary to suppose it enlarged in order to compare the direction of the incident with that of the refracted ray.

68. To do this completely we must be acquainted with the law, according to which the attractive force increases, as the luminous particles approach the refracting surface. Of this law we know nothing, except that it must produce a very rapid increase of velocity, and an equal increase at equal distances, through the whole extent of the surface. Happily these general data are sufficient to conduct us to very important results.

In order to represent the increase of attractive force in the most general manner, let us divide the space in which it is sensible into an infinite number of very thin zones by the lines $c\,d$,
Fig. 59.

ef, *g h*, &c., parallel to the surface of the medium and to the first limit *a b*. Then let us suppose that in each of these zones the intensity of the attractive force is sensibly constant, so that it shall only increase in passing from one zone to another. Continue this construction into the interior of the medium as far as to the second limit, where the attractive force ceases to be sensible. This being done, if we do not establish absolutely any relation between the successive values of the force for these different zones, there can be no law so general that it may not thus be represented. The conformity will be the more perfect, the more we multiply the zones, and it will be entirely so, by supposing their number infinite. We can then employ this supposition for the purpose of representing the progress of the attractive forces; and if we thence deduce results which are independent of the number of zones, we may be certain that they belong also to the attractive forces themselves, whatever be the law by which they are governed.

The whole is then reduced to considering what takes place, when a particle infinitely small, projected into void space with a certain direction and velocity, traverses a zone comprehended between two parallel planes, where it is urged by a uniformly accelerating force. For if we resolve this problem for the first zone, we shall be able to calculate the direction which the luminous particle will have acquired there, together with its increase of velocity. We shall then proceed to the calculation for the second zone with these data, and thence to the third, and so on through the whole thickness in which the attractive forces are sensible.

This problem is precisely that of the motion of projectiles in void space, supposing them acted upon by gravity alone. By resolving it we find that for the same substance the ratio of the sine of incidence to the sine of refraction, is constant under all possible inclinations, as experiment has shown us. We find also, that this ratio is the same as that of the velocities of light, after it has penetrated the body to a sensible depth and before entering it. This initial velocity is always more feeble than the other, if the ray passes from a vacuum into a material substance, so that in this case it is accelerated by being refracted.

Finally, the analysis, by discovering all the particulars of the phenomenon, enables us, by experiment, to ascertain, if not the

absolute value of the attractive force, at least a quantity proportional to it in each body. This quantity is the square of the ratio of refraction diminished by unity, and divided by the density of the refracting substance. Newton gave it the name of *refracting power.* Its estimation supposes that the luminous particles, in their passage through bodies, are acted upon only by forces peculiar to ponderable material particles, while such imponderable and intangible principles as those of electricity and caloric, do not contribute either directly or indirectly to produce or modify this action. The only means of judging whether this supposition is correct, is to compare the results derived from it with those deduced from observation.

69. Let us now consider what takes place when the luminous
Fig. 60. particle approaches the surface of emergence $A_2 B_2$, which, for the sake of simplicity, we will suppose parallel to the first surface. The attractive forces, exerted by the body near these two surfaces, will have the same extent of action. Let us then represent, by the parallels $a_2\, b_2$, $a_3\, b_3$, the interior and exterior limits where these forces cease to be sensible. This being done, while the luminous particle is approaching the interior limit $a_2\, b_2$, it will continue to move in a straight line with the velocity derived from refraction; but when it has passed this limit it will begin to be attracted again towards the interior of the refracting body, by forces varying with the successive zones through which it passes. In order to calculate the effects which these forces will produce upon it, it is only necessary to decompose its velocity into two others, the one parallel to the surface of emergence $A_2 B_2$, and the other perpendicular to it. Now it is evident that the action of the attractive forces will not change the first velocity which is perpendicular to their direction; but it will tend constantly to diminish the normal velocity to which it is parallel and opposite, since this velocity tends to bring the luminous particle without the body, while the attractive forces have a contrary tendency.

Consequently, from the instant the action of these forces begins, the path of the luminous particle will be in a curve convex towards the surface of emergence; but on account of the similarity of position which we have supposed between this surface and the surface of entrance, the actions experienced by the particle will be exactly equal, so that in approaching the second,

surface it will not lose what it had acquired in departing from the first. It will therefore reach $A_2 B_2$, with the same velocity which it had at AB; and then, passing this limit, it will continue to be affected by the action of the medium ABA_2B_2, which will retard it, and will tend to draw it inward, but with an energy continually decreasing in proportion as its distance becomes greater; until at length this action entirely ceases to be sensible, when the particle has reached $a_3 b_3$, the exterior limit of the attractive forces, after having gradually lost all the increments of velocity which it had acquired at its entrance into the refracting medium. Then the particle, being no longer influenced by the attraction of this medium, will continue to move with its primitive velocity, in the direction of a straight line, tangent to the last element of the curve it has described; and this tangent, on account of the parallelism of the two surfaces, will be parallel to the primitive direction of the particle before it entered the refracting medium.

70. Let us now consider the case where the surface of emergence A_2B_2, instead of being parallel to the first surface, is inclined to it at a certain angle. Then the limits of the action of the medium will still be near this surface, and we can likewise represent them by the straight lines $a_2 b_2, a_3 b_3$, drawn at the same distance as before. Moreover, the succession of actions exerted by the medium, at different distances from the surface of emergence, will also be the same. But the normal velocities which these actions will have to oppose will be different; for, upon repeating here the decomposition which furnishes these velocities, it will be seen that they become more feeble in proportion as the direction of the ray approaches the surface of emergence. If their value is still such that the attractive forces are not sufficient to destroy it, the luminous particle will traverse all the zones where these forces are sensible, will pass their exterior limit, and having regained all its primitive velocity, will move off in a straight line in such a manner that the sine of its emergence will be to the sine of its interior incidence in a constant ratio, and this will be the same that it has at the surface of entrance. But it is also possible that the luminous particle may reach the surface of emergence with such an inclination, that its normal velocity, which alone tends to make it emerge, will be entirely destroyed by the continued and increasing attraction of the medium, before it has passed this surface; or there may remain sufficient Fig. 61.

velocity to make it pass, and then this remainder may be destroyed by the retarding action of the medium before it has attained the exterior limit $a_3\ b_3$, where this action ceases to be sensible. In both cases, the luminous particle, in proportion as its normal velocity is diminished, will yield to the action of the other component whose intensity does not suffer any diminution, so that its path will be bent more to the surface of emergence to which this component is parallel. The moment this component comes to act alone, the normal velocity being destroyed, it will cause the luminous particle to describe a small element of a straight line parallel to the surface of emergence; after which the attractive action of the medium, experiencing no obstruction, will draw the luminous particle inward by a course symmetrical with that by which it approached the surface of emergence, and will communicate to it in a contrary direction the same degrees of normal velocity which it had before destroyed, until the particle, having reached the interior limit $a_2\ b_2$, where the attractive forces cease to be sensible, will continue its course in a straight line, with the same velocity it had before entering the strata of variable attraction, and in a symmetrical direction; so that the angles of reflection and incidence will be equal.

71. This analysis makes known the origin of the change from refraction to reflection, which is actually observed in certain circumstances at the second surface of bodies; and teaches us, moreover, that this change is not limited to a single incidence, but may be distinguished into three cases. 1. When the reflection takes place in the substance of the refracting medium between the interior limit of the attractive forces and the surface of emergence; 2. When it takes place at the surface of emergence; 3. When it takes place between this surface and the exterior limit of the attractive forces, the luminous particle emerging first from the refracting medium, and afterwards entering it. This last case is that in which the angle of emergence, calculated according to the ratio of the sines, becomes equal to 90°; it is also that which determines the interior incidence the nearest to the normal, under which the total reflection takes place. We shall see, hereafter, that these different limits become very manifest from observation, although the spaces in which they are comprehended are altogether insensible; insomuch that the ray appears

to be broken suddenly and at the same point in these different cases.

Hitherto we have considered the bodies subjected to experiment, as contiguous to a void; we shall now see what takes place when the light passes from one refracting medium into another. It is easy to combine these two phenomena; for it is sufficient to remark that in this case the luminous particle is attracted, not only by the medium which it leaves, but also by that which it is to enter.

Let M be the first medium, M' the second, and $A'B'$ their common surface. While the particle is at M, so far from the surface $A'B'$, that the attraction exerted by the first medium is equal on all sides, and that of the second still insensible, it will move in a straight line with the constant velocity which the action of the first medium has impressed upon it. But when it has arrived within the limits of distance where the intensity of the attractive forces becomes variable, it will begin to experience simultaneously the action of the two media, drawing it in contrary directions; that of the medium it has left tending to retard it, and that of the medium to which it is approaching having a tendency to accelerate it. Accordingly it will be influenced only by the difference of these actions, which are both directed perpendicularly to the common surface $A'B'$. Hence we see that if the medium which it is about to enter, acts at an equal distance with greater force than the other, it will always enter, and after passing, in a curvilineal direction, beyond the interior limit $a_3\, b_3$ of the attractive forces, it will at length go on in a straight line with a constant velocity. But if, on the contrary, the first medium acts more strongly upon the ray than the second, we may consider the luminous particle as moving in a medium, whose action will be simply the difference of action of the two media, and consequently retarding, $A'B'$ becoming the surface of emergence. Then we shall find all the cases of emergence and of interior reflection which we have already considered. Thus the incidence may be such that the particle shall be refracted and afterwards move in a straight line into the second medium; but it is likewise possible that the refraction may be changed into reflection; and here, as when the particle emerged into a vacuum, the reflection may take place, either in the interior of the first medium, between the interior limit $a_1\, b_1$ of the Fig. 62.

attractive forces and the common surface $A'B'$; or at the surface $A'B'$ itself; or lastly, in the interior of the second medium, even to the second limit a_3 b_3 of the attractive forces. But generally, when the ray passes from one medium to the other, whichever be the medium that acts most strongly upon the light, the ratio of the sine of incidence to the sine of refraction will be constant, and the reciprocal of the ratio of the velocities in the two media.

72. Let us now endeavour to realize these results by experiment, and see whether they are conformable to theory. In the first place, as to the constancy of the ratio of refraction and its value, we can easily verify them in the following manner. Having measured the deviation effected by a prism under a determinate incidence of the luminous rays, extend over one of its surfaces a stratum of some transparent substance, rendered throughout of the same thickness by being pressed with polished glass, or by being ground and polished. It will be found that the addition of this stratum to the prism does not alter the observed deviation, the incidence remaining the same; wherefore the ratio of refraction which the light experiences in penetrating the stratum, is exactly counterbalanced by that which takes place at the second surface where it touches the glass, so that it penetrates the glass in the same manner as if it had entered it directly. This is at once a consequence and a verification of the properties deduced theoretically.

In order now to observe the progressive effect of the incidence upon the interior reflection construct a prismatic vessel $AABB$, whose anterior and posterior faces, formed of glass, may be gradually inclined the one to the other, and remain exactly fitted to two parallel planes of copper, in such a manner as to form a prismatic vessel of a variable angle. Then, having filled this vessel with water or any other liquid, dispose vertically the anterior face BB, and direct perpendicularly upon it a fixed horizontal ray SI, obtained from a heliostat. The ray thus penetrating the liquid with a perpendicular incidence, will continue its course in a straight line till it reaches the second surface of the prism, where it will experience, in general, a partial reflection, which will carry a portion of it into the interior of the liquid, and thence out to O'' through the horizontal surface. The rest, experiencing the action of the refracting forces, will come out refracted in the direction $I'O$, if it is capable of emerg-

ing; and if not, this also will be drawn into the interior, making the angle of reflection equal to the angle of incidence, and will go to join in O'', the portion partially reflected. Then, in order to follow out the phenomenon, render the second face AA vertical, and having formed a liquid plate with parallel faces, the ray will be seen to pass on without any deviation whatever, as if there were no prism interposed. But as the angle is opened a little the refraction will begin to take place, the emerging ray being turned upward. This deviation will increase in proportion as the angle is enlarged, which will bring the interior incidence nearer to a coincidence with the second surface. We shall at length arrive at a point in which the obliquity will Fig 65.
be such, that the emergence will be hardly possible, the ray at Fig. 66.
its departure grazing the surface of the second glass. Beyond this term the ray will no longer emerge, but will be entirely reflected inward. This will take place when the sine of the interior incidence, reckoned from the normal, is equal to unity divided by the ratio of refraction. Then the total reflection will continue to take place inward, under every enlargement of the angle of the prism, this enlargement bringing the interior incidence always nearer to a coincidence with the second surface. These details are strictly conformable to the results furnished by the theory of the attractive forces. Fig. 65.

73. One circumstance must here be mentioned which will be useful hereafter; at the moment when the emergence is on the point of becoming impossible, it does not cease suddenly and at once for the whole ray; but the part of this ray which produces the sensation of red is the last to disappear. Such indeed must be the result, if, as we have already remarked, this portion is composed of rays less refrangible than the rest, and whose ratio of refraction consequently differs less from unity.

It may be asked how it happens that a certain number of particles, under all incidences undergo an interior reflection, even when their velocities, decomposed perpendicularly to the refracting surface, are sufficient to make them emerge. It arises from this, that beside the velocity of translation, which alone we have here considered, these particles are affected also by particular physical circumstances to be made known hereafter, which favor the action exerted upon them by the reflecting forces of the second [illegible] an action from which the other particles are free, being [illegible] yield to it.

74. When a prism of a variable angle, like that we have described is not at hand, the effect of interior reflection may be observed in a very simple manner. Place a triangular prism
Fig. 67. ABC, between the light and the eye in such a manner that the incident rays SI, entering by the first face AB, may be reflected at the base BC, and then pass out through the posterior surface AC. If we would attend to all the particulars of this phenomenon, the eye must first be placed sufficiently high to allow the objects situated below the base BC to become visible by refraction through the angle BCA. In this position the rays SII', which come from objects situated above AB, are refracted also in I', and emerge into the air above the prism, with the exception always of the feeble portion which experiences in I' a partial reflection. But if the eye be lowered a little there will be found a position in which the objects placed below BC cease to be visible by refraction; and, on the contrary, the exterior objects, situated above AB, are represented on the base BC, as in a mirror. It is because the refracted rays II', becoming, in this case, too oblique to BC to be capable of emerging into the atmosphere, are all reflected inwards; and as no ray, coming from the objects situated under BC, can, upon being refracted by the prism, depart so far as these from the normal $I'N'$, none can any longer reach the eye. Thus, when this total reflection has once commenced, it will continue under all the greater obliquities.

75. In order to fix the exact limits where these objects disappear, consider $N'I'I''$ the greatest possible angle of refraction for the rays which enter the prism by the base BC; and draw the emergent ray $I''O$, resulting from the second refraction through the face AC. Then, if we draw through the vertex of the angle C, the line CO' parallel to $I''O$, it is manifest that no ray coming through the face BC, can enter the angle $O'CC'$, comprehended between this straight line and the prolongation of the base of the prism. Accordingly, while the eye is situated within this space all the objects below BC will be invisible by refraction.

76. It is to be remarked that the phenomena of total reflection can never be observed when we look through the same face of the prism at which the rays enter, as is shown in figures 68 and 69. For, if the prism be turned as in the first of these figures, the refracted ray II' can never be totally reflected at the second

surface CB; and if, on the other hand, the prism be turned as in figure 69, the reflected ray $I'I''$ will cease to be capable of escaping through the first surface AB before its total reflection at the second AB begins to takes place. Fig. 69.

Let us recur now to the mode of observation employed above; Fig. 67. but instead of leaving the whole base BC in contact with the air, put a drop of water on one of its points E. Then, as the prism Fig. 70. is turned slowly about its axis, the eye being placed at O, so as to receive the reflected rays SII' coming from the clouds, it will be seen that the total reflection begins to take place on every part of the base BC, which is contiguous to the air, before taking place at the point E where the drop of water is placed; so that this remains some time visible through the thickness of the prism by means of the rays which emanate directly from it. As the eye is lowered towards the base BC, to render the rays more oblique, the total reflection gradually begins at E, and the drop disappears.

77. All these phenomena are the necessary consequences of the theory. When the refracted rays IE fall upon the base BC with an incidence nearly perpendicular, they do not experience a total inward reflection, even at the point of the base contiguous to the air. Reciprocally every ray coming with this incidence from objects situated below the base of the prism, enter it and come to the eye. But when, upon lowering the eye, the refracted rays IE become more and more oblique to the base BC, we at length reach a point where the action of the air in contact with this base is not strong enough to transmit them; then, they will be totally reflected, after departing from the prism, provided they do not exceed the exterior limit of the attractive forces of glass and air. But the incidence at which this phenomenon takes place, is not the same for the point E where the drop adheres, because the attractive action of the water upon light is much stronger than that of air; which renders the normal forces, tending to make the luminous particles emerge, much more powerful. It is thus necessary to lower the eye more towards the base BC, in order that the interior reflection may take place at the point E where the drop is situated; and it is only when such an inclination is effected that the drop entirely disappears. At this instant there is a certain number of rays SI, coming from external objects, which pass through the drop at an infinitely small distance from the

base of the prism, and which being then drawn into the interior of the prism by refraction, reach the eye. When the drop is of a transparent liquid, these rays in their short passage through it are not sensibly weakened. Their brightness in this case prevents us from distinguishing the feeble light which is still sent to the eye by the infinitely thin liquid stratum embraced by the attractive forces. Accordingly the drop disappears.

The limit would manifestly be different if the substance applied to the base *BC* were opaque, even though its refracting force were otherwise the same as that of water. For then the rays *SI*, which must penetrate it in order to be reflected would be arrested by its opacity. In this case, we should begin to see the total reflection only at the moment when the refracted rays *IE* are reflected at the limit of the two media; which requires an interior incidence approaching more nearly to this surface. Although the difference of thickness by which this case of reflection is distinguished from the preceding, is infinitely small, or at least, incapable of being appreciated by our senses, yet its influence upon the angles of incidence and reflection is very perceptible. If, for example, the substance applied to the base of the prism be bees-wax, which can be rendered alternately transparent and opaque, by being subjected to the flame of a candle and then suffered to cool, it will be found that the total reflection, in this last case, will begin at an incidence at least eight degrees nearer the reflecting surface than in the other. Laplace was the first who pointed out this difference of limit required for interior reflection in transparent and opaque bodies. He has afforded the means of calculating the incidences in both cases, according to the theory of attractive forces; and Malus has realized these results in the experiment which we have described; adding at the same time the exact measure of the angles, which prove that the limits of incidence, given by observation, agree exactly with the theoretical ones. This agreement, and the constancy of the ratio of refraction, are two strong circumstances in favor of the hypothesis of the emission of light, especially when we consider by what secret links these two results are connected together.

78. We are indebted, moreover, to Malus for another experiment, which, without being susceptible of measurement, affords a striking example of the action of the attractive forces in the

phenomenon of total reflection. He placed upon the base of a prism of crown glass a drop of ink, which being at first transparent, permitted total reflection under certain angles of inclination. But gradually, as this drop increased in density by drying, greater degrees of obliquity were necessary in order to make it disappear; and at length becoming entirely opaque, it rendered the phenomenon impossible by reason of the great refracting force of the metallic particles which enter into its composition. If the glass of the prism had been of a kind possessing greater refracting power, perhaps the reflection would have been possible; and then the incidence at which it took place would have determined the ratio of refraction of the ink, though it had become impermeable to light. In general, it appears that we can determine, by this method, the ratio of refraction even of opaque bodies, when we can apply them to the second surface of a transparent prism, possessing a greater refracting power than the bodies themselves. But it is necessary carefully to calculate the total reflection as it begins to take place at the common surface of the body and the prism. This ingenious process was suggested and applied by Dr Wollaston; but it was Malus who first remarked the necessity of a different calculation for the results of experiment, according as the body applied is opaque or transparent; and this he did from the formulas for the two reflections given by Laplace. We hence derive a simple method of finding very nearly the ratio of refraction of a given liquid, when the quantity possessed is quite small. We have merely to apply a drop of it to the second surface of a prism, and observe if it disappears sooner or later than the drops of other liquids whose ratio of refraction is known.

79. Having thus explained the methods of determining by experiment the refracting powers of all transparent bodies, and also those of opaque bodies, when they can be brought into contact with a transparent body of a greater refracting power than themselves; it remains to bring together the results obtained in this way for different classes of substances, in order to see whether we can discover any relation between their refracting forces and their chemical composition. This is the purpose of the following table formed by Newton.

Refracting Substances.	Ratio of the sine of incidence to the sine of refraction for yellow light. n.	Values of the quantity. $n^2 - 1$.	Density of the refracting substance. ρ.	Its refracting power. $\frac{n^2 - 1}{\rho}$.*
False topaz (sulphate of barytes)	23 to 14	1,699	4,27	3979
Air	3201 to 3200	0,000625	0,0012	5208
Glass of Antimony	17 to 9	2,568	5,28	4864
Selenite (sulphate of lime) . . .	61 to 41	1,213	2,252	5386
Common Glass	31 to 20	1,4025	2,58	5436
Rock crystal	25 to 16	1,445	2,65	5450
Iceland spar	5 to 3	1,778	2,72	6536
Common salt (muriate of soda) .	17 to 11	1,388	2,143	6477
Alum (sulphate of potash) . . .	35 to 24	1,1267	1,714	6570
Borax (borate of soda)	22 to 15	1,1511	1,714	6716
Nitre (nitrate of potash)	32 to 21	1,345	1,9	7079
Dantzick vitriol (sulphate of iron)	303 to 200	1,295	1,715	7551
Oil of vitriol (hydro-sulphuric acid)	10 to 7	1,041	1,7	6124
Rain water	529 to 396	0,7845	1,00	7845
Gum Arabic	31 to 21	1,179	1,375	8574
Alcohol	100 to 73	0,8765	0,866	10121
Camphor	3 to 2	1,25	0,996	12551
Olive Oil	22 to 15	1,1511	0,913	12607
Linseed Oil	40 to 27	1,1948	0,932	12819
Spirits of Turpentine	25 to 17	1,1626	0,874	13222
Amber	14 to 9	1,42	1,04	13654
Diamond	100 to 41	4,949	3,4	14556

From inspection of this table it appears that substances of very different densities may have equal refracting forces; and also that a substance of less density than another may yet possess a greater refracting power. Accordingly, as we before observed, the action of bodies upon light does not depend upon their density alone, but also upon the chemical nature of their particles. It appears, moreover, that the substances whose refracting power is greatest, are in general, the resins and oils; and as that of distilled water is little inferior to them, it is natural to conclude that there must be in water some inflammable principle analogous to that of which the resins and oils are composed. The same conclusion must be extended likewise to the diamond whose refracting power is still greater. These striking suggestions did not escape the sagacity of Newton; and he did not hesitate to make them known. For this great man, who practised the greatest severity in his experiments, and the most cautious re-

* The numbers belonging to this last column have been multiplied by 10000 to avoid decimals. In the first column, the modern names are subjoined to those used by Newton. The great weight of the stone which he calls *false topaz*, leaves but little doubt that it is the sulphate of barytes.

serve in his conjectures, never feared to follow out the consequences of a truth however far it might lead him.

If we would discover what this principle is which is common to the oils and resins, and which gives them so powerful an action upon light, we have no better means of doing it than by determining the refracting powers of gaseous substances; for, since almost all bodies with which we are at present concerned, are composed of gases combined together, we shall thus have the advantage of surveying them in their most general elements. This, as before mentioned, Arago and myself have done. In what manner the observations were made and what were their immediate results, we have already stated; it remains now to make known the refracting powers thence deduced.

Refracting Power of the Gases for the Temperature of 32°, *and the pressure of* 29,92, *deduced from a collection of Observations.*

Gases.	Density, that of atmospheric air being unity.	Value of $n^2 - 1$.	Refracting power, that of air being unity.
Atmospheric Air . .	1,00000	0,000589171	1,00000
Oxygen	1,10359	0,000560204	0,86161
Azote	0,96913	0,000590436	1,03408
Hydrogen	0,07321	0,000285315	6,61436
Ammonia	0,59669	0,000762349	2,16851
Carbonic Acid . . .	1,51961	0,000899573	1,00476
Carburetted Hydrogen	0,57072	0,000703669	2,09270
Hydrogen more carburetted than the preceding	0,58825	0,000630300	1,81860
Muriatic Acid Gas .	1,24740	0,000879066	1,19525

All the densities collected in this table are the results of our own experiments. The values of the refracting powers would be somewhat different, if we were to substitute the densities given by MM. Dulong and Berzelius.

From inspection of this table, it appears that the refracting power of hydrogen very much exceeds that of any other gas, and even that of any other substance hitherto observed. This element exists in great abundance in resins, oils, and gums, where it is united with carbon and oxygen; and is accordingly that which gives to these substances the great refracting power which Newton so justly remarked. This influence of hydrogen

these humours are in a short time replaced; if the pupil becomes closed up, we can make an incision in the iris and thus form an artificial pupil. Finally, if the crystalline itself becomes opaque, and it is necessary to remove it in order to open a passage for the light, we can still see, without its assistance; only the distance of distinct vision is greatly increased, as we should naturally expect, after taking away from the instrument a converging lens; but we supply the defect by placing before the eye a converging glass of a suitable curvature; and then we can see almost as distinctly as with the natural crystalline unimpaired. This stability seems to be a general characteristic of all our organs of sense.

224. Hitherto we have considered only the instrument of vision. Its connection with the sensation, presents a mystery still more inexplicable. All we know is, that the impression produced upon the retina is transmitted to the optic nerve, and by that to the brain. The circumstance that we have the sensation of objects erect, although we receive the image of them inverted, is not surprising. The image, which is the cause of the sensation, must not be confounded with the sensation itself. The rays refracted by the humours of the eye, have, when they arrive at the retina, directions quite different from those which they had when they reached the cornea; yet, in thought, we always refer the object to the prolongation of this primitive direction. We do this, because the experience of our whole life has taught us that the object is found in this direction. This result of experience becomes associated with the sensation, as an invariable consequence, and the mind draws the inference instantaneously. Accordingly, a deception is practised, when the same artificial indications are exhibited, without the intervention of real objects. Thus objects, seen by reflection from a plane mirror, appear to be situated beyond its surface, although reason and experience, but an experience subsequent to the first lessons of our senses, apprise us of the error. This principle offers a simple and natural explanation of all the optical illusions, which are produced by glasses and mirrors.

225. When we look at objects with only one eye, and are not previously apprised of their distance, we suppose them at the vertex of the luminous cone, which has the opening of the pupil for its base, and for its vertex some point of the object. Ac-

cordingly, this estimate is only so far just, as the objects are sufficiently near to allow the angle of divergence of the rays which form the cone, to be sensible. When we look with both eyes at once, the same principle leads us to suppose the object placed at the point of meeting of the pencils which arrive at the two eyes, and which is the common vertex of the cones of which we have been speaking. Then the basis of measurement is the space between the eyes. It is therefore greater than in the preceding case. Accordingly, this estimate is much more correct than the other, as we may easily assure ourselves by observing the difficulty of threading a needle when we look at it with only one eye, while nothing is easier when we look at it with both. But in order to see images single, it is necessary that they should fall upon corresponding parts of the retina, where we are accustomed to see them when they come from the same object. If we press the eye a little sideways with the finger, we immediately see two images, one at the same place with the former, and the other on one side of it in the direction in which the pressure has carried the eye. But if we keep up the pressure for some time, this secondary image becomes faint and finally disappears, and we again see the image single at the same place as before.

226. Experience teaches us that such pressures, when sudden and violent, produce tremors in the optic nerve from which results the sensation of light. This sensation may also be excited or destroyed by contrast. If, for example, the eye has been fixed for some time upon an extended space of a uniform colour, it seems afterward to abstract this colour, when it is directed to any other object; and we thus see a spot on this object, the colour of which is *complementary* of that upon which the eye had been resting; that is, it is composed of those rays from the object, which did not make a part of the first colour. These appearances, produced by contrast, are designated by the name of *accidental colours*.

227. Sometimes also, we observe in luminous objects, certain deceptions of another kind; namely, stripes tinged with the colours of the rainbow, or bright halos of divergent beams. These effects are produced by the decomposition of light in the small humid drops, which are accidentally lodged between the

is eminently exhibited in ammonia, which is composed of hydrogen and azote. The refracting power of this gas is double that of air, and therefore, according to the first table, exceeds that of water.

But let us go further; and, since each substance seems to carry with it its peculiar character in all the combinations into which it enters, and to retain, to a certain extent, the force with which it acts upon light; let us endeavour to estimate under this view the influence of the constituent principles which belong to any given mixture or composition.

80. If we should attempt to discover these relations for any other substance but light, we should meet at once with insuperable difficulties, arising from the combination and degree of condensation of the constituent principles; for, although the chemical action is exerted only at very small distances, yet these distances admit of being compared together; so that the greater or less distance between the particles cannot fail to alter the intensity of the action. These variations, further modified by the figure of the particles, must render the ratios between the compounds and their elements, extremely complicated; and though we cannot estimate the effects thus produced, it is manifestly on account of them that the two have not the same properties. But, according to the most probable theory of light, the extreme minuteness of its particles, compared with the distances by which the particles of bodies are separated, must render the effect of small changes in these distances less sensible. Accordingly the refracting powers of bodies must differ little from those of their constituent elements, especially if these powers are to be ascribed simply to the action of the material, ponderable particles, and if their energy is not modified by the greater or less abundance of imponderable principles, united with them, which the act of combination almost always changes.

In order now to determine what must be the influence of each element, it must be remembered that the refracting power of a body is a quantity proportional to the sum of the attractive forces which it exerts upon light at different distances, and with a density equal to unity; this sum being reckoned from the instant the luminous particle begins to be sensibly attracted by the body, to the instant it arrives at the surface. If then we suppose that the proper action of each element is not altered in

the combination, it will follow that at each distance, taken within these limits, the total attraction experienced by the luminous particle, will be equal to the sum of the attractions exerted upon it by the different elements composing the attracting body; and the part of the effect due to each element, will be proportional to the product of its proper refracting power by its mass, that is, by the ponderable quantity of this element which enters into the combination.

81. Let us, in the first place, try this law in those simple cases in which there is little or no condensation. Atmospheric air will afford a convenient example. It is known that this air when dry, contains 0,21 by bulk of oxygen gàs; the rest is a mixture of azote, carbonic acid, and perhaps other gases in imperceptible proportions. For the sake of simplicity, we will take into consideration only the azote and carbonic acid; and will suppose 0,784 of the first, and 0,006 of the second, these quantities being always reckoned by bulk. I adopt these proportions, because they agree very nearly with the results obtained by analyzing atmospheric air, and because they satisfy the values of the densities determined by experiment. Now in order to find the ponderable quantities of each element which enters into the volume 1, each fraction of this volume must be multiplied by the density of the gas of which it is composed. We shall thus have

Oxygen . . .	$0{,}210 \times 1{,}10359 = 0{,}231754$
Azote	$0{,}784 \times 0{,}96913 = 0{,}759798$
Carbonic acid .	$0{,}006 \times 1{,}51901 = 0{,}009114$
	$1{,}000666$

This sum is nearly equal to unity; and that is what it ought to be, for it must express the specific gravity of the mixture, that is, of atmospheric air, which we have taken for unity. The error is like those that occur in observations of which no account is to be given. By neglecting it, the results which we have calculated will be precisely the ponderable quantities of the three elements which compose a mass of atmospheric air equal to unity. It remains then to multiply each of these by the refracting power which corresponds to its nature, and we find

Oxygen	0,199682
Azote	0,785693
Carbonic Acid	0,009157
Refracting power of the composition	0,994532

The sum of these numbers expresses the refracting power of atmospheric air, deduced from its constituent elements. To be perfectly exact, it ought to be equal to unity; the error is equal to 0,005468, or about five thousandths of the total value. It would not amount to three tenths of a second in the elevation of the pole at Paris; and this difference may arise from errors almost inevitable in making experiments; for the preceding result, depending on the specific gravity of the gases, their purity, and the refractions they produce, is connected with a great number of operations which are liable to error.

Accurate and numerous analyses of atmospheric air, made in different climates, and under circumstances the most various, have shown that its ponderable principles are throughout the same, and that they exist always in the same proportions. It follows from this, that the refracting power of atmospheric air is also the same throughout the earth, since it is determined by the partial refracting powers of its constituent elements. Accordingly, the tables of refractions calculated for one latitude, may be employed in all climates, regard being had to the variations of density produced by change of pressure and temperature. As to the difference which may arise from the moisture of the atmosphere, we shall show hereafter that it is sensibly nothing, and that it may be wholly neglected.

82. Let us now consider the cases in which the constituent elements are united by actual combination; and to proceed gradually, let us begin with affinities that are not very powerful. We have a very suitable subject in ammoniacal gas, composed of hydrogen and azote, whose combination does not prevent its preserving the gaseous state. In this example, as in the preceding, according to the analysis of M. Berthollet, the result is such, that if the compound had not been analyzed, and we had hardly known the nature of its elements, its refracting power would have indicated exactly its composition.

Proceeding now to more intimate combinations, we calculate in the same manner the refracting power of water from its constituent elements, hydrogen and oxygen. According to very accurate experiments made by MM. Guy-Lussac and Humboldt, water is composed of two parts by bulk of hydrogen to one of oxygen; which, reduced to weight, gives for 1000 parts, 117 of the first to 883 of the second. Each of these quantities being multiplied by the value of the refracting power belonging to it, the sum of the products gives 1,53545 for the refracting power of water, that of air being unity. The refracting power actually observed by Newton, is found to exceed this by about $\frac{1}{8}$ of its total value. Thus oxygen and hydrogen gas, being condensed into water, exert upon light an action sensibly stronger than when in a state of simple mixture. The same experiment made upon other compounds gives similar results; that is, the change from a gaseous to a compact state, always produces a sensible increase of affinity. But this is particularly remarkable in the diamond, which, according to experiment, has a considerable refracting power, although from the most accurate analysis, it appears to consist entirely of carbon, a substance which in a gaseous state, exerts only a very feeble action upon light; at least if we may judge from carbonic acid, of which it is one of the constituent elements.

Still the refracting power, deduced from chemical composition, will be found to differ very little, if at all, from the truth, when the state of the body undergoes only slight changes. The rule may indeed be employed with perfect safety in the case of mixed liquids, even those which exert a very strong action upon each other, as water and alcohol. We have seen that the mixtures of the gases presented a similar agreement.

From what has been said, it seems probable that the refracting power of water in a state of vapour, differs very little from water in a liquid state; and hence we are able to calculate the influence which its presence in the air has upon atmospherical refractions. We thus find that the excess of refracting force of this vapour over air, is almost exactly compensated by its inferior density; so that on the whole, the refraction produced by masses of dry and humid air are sensibly the same, the pressure and temperature being supposed equal in the two cases. Indeed, this equality has been proved by experiment, and has been found

so exact that it was impossible to perceive any difference in the two cases. We hence deduce a practical principle of great importance to astronomers, which is, that their observations are independent of the hygrometric state of the air; for, if moisture had any appreciable effect, it would render the calculations in astronomy much more complicated. Nevertheless, by a mode of observation still more delicate than the one here made use of, M. Arago has ascertained that aqueous vapour has a refracting power a little less than that of water in a liquid state; and that in this respect there is a difference also in all other vapours, when compared with the liquids from which they are derived. Happily this difference is so small in the case of water, that if strict accuracy require that we should take account of it, still experiment shows that it may be neglected.

83. In like manner we learn that change of temperature produces no perceptible change in the refracting powers of the gases; for the deviations observed in summer and in a room where the moisture and temperature are artificially varied, agree exactly with the results of calculation from refracting powers deduced from observations taken in winter. It is probable, however, that we should find a difference in this respect, if we were to operate at such degrees of cold as to admit the reciprocal action of the gaseous particles to become sensible by their approach. It is also possible that in a liquid condensed by cold, the attractions resulting from the configuration of its particles might modify sensibly their action upon light. Perhaps we may attribute to this cause the curious fact observed by M. Arago, that the refraction of water in a liquid state increases in proportion as its temperature is reduced, even when its density is diminished by the dilatation it experiences below the temperature of 39°.

84. Hitherto we have considered the motion of light in homogeneous media; but we may likewise conceive of media of different strata and composition, with a view to discovering what takes place when a ray of light traverses them. This problem is solved by means of the same principles which guided us in calculating the course of luminous particles as they approach the surface of bodies, or as they penetrate their substance to a small depth. We divide the medium, traversed by the light, into such a number of strata as to allow the density and composition of each to be regarded as uniform. Then, in each

of these strata, the path of the luminous particles may be considered as a parabola, whose direction is determined by the actual intensity of the refracting power; and, continuing the calculation from stratum to stratum, we can ascertain successively the path of each luminous particle from the direction in which it is projected.

Suppose *ABCD* such a medium, composed of horizontal strata, Fig. 71.
whose ratio of refraction is at first constant for a certain height, and then goes on decreasing by insensible degrees, till at length it again becomes constant, but less than before. We shall have an instance of such a medium, if we conceive in the same vessel a thick stratum of sulphuric acid covered by a stratum of pure water, a disposition easily effected if we pour the water in first, and then introduce the acid by a glass tunnel, the end of which, drawn out into a narrow tube, is directed to the bottom of the vessel. The acid being heavier than the water, extends under its surface and raises it in proportion as its quantity increases. Now these substances having a considerable affinity for each other, tend to combine together, and the combination in fact takes place in the lowest strata of the water which lie immediately upon the acid. But by reason of their plane figure, the attraction which they exert upon each other, as well as the difference of their specific gravities, prevents them from changing places; and the strata of the two substances which are in contact, being always the same, the combination can only be propagated from one to the other very slowly. Hence in spite of the strong affinity between water and sulphuric acid, if we do not make use of a very large vessel, we often find that after a whole day the upper strata of the water remain unmixed with the acid. Here then is a medium composed of strata that are parallel, heterogeneous, of different refracting powers and powers decreasing with the height. For the ratio of refraction of concentrated sulphuric acid exceeds that of pure water, and it carries this property into the strata with which it enters into combination. Let us now suppose that the experiment is made in a rectangular vessel of thin glass, in order that its vertical sides may present transparent parallel surfaces which will not change the regularity or the course of the luminous rays. Conceive now a radiating point situated at *A*, on one of the sides of the vessel, at such a height that the strata of acid which correspond to it, shall

not be sensibly mixed with water. Then this point will be capable of sending through the acid a horizontal ray, which will pass directly through the thickness of the vessel and may be received by an eye situated on the other side at *O*. But there may also arrive at *O* another ray, which, proceeding from the same point with the first, is first directed towards the upper strata in which the water is mixed with the acid. For, meeting in these strata a refracting power successively decreasing, the attraction of the lower strata will continually draw and bend it towards them; and if the inclination be properly chosen, this attraction will be sufficient to change it entirely, and cause it to turn towards the bottom in such a manner as to traverse anew the same strata in a contrary direction, and descend towards the point *O*, to which the first ray was directed. This effect actually resembles what takes place at the point of contact of two media of different refracting powers, except that, in this last case, the interior reflection is effected in a very small space, and is rapidly determined by the diminution which the attractive forces undergo as we depart from the common surface; while in the mixture of water and acid, the decrease of the attractive forces is rendered very slow by the gradual proportions of the combination. But it equally happens that at the point *O*, under certain limits of inclination, we can perceive two images of the radiant point, the lower one by its direct path, the upper one by its curvilinear path. To realize these consequences, paste to one of the sides of the vessel, a little below the intermingling strata, a small horizontal strip of paper, containing some writing. Then, on placing the eye the other side of the vessel, at nearly the same height, and attempting to read these letters, we shall see two distinct images, of which the lower one will be upright, and the upper one inverted. This ingenious mode of making the experiment was devised by Dr Wollaston.

A similar phenemenon will be exhibited if, in the summer season, we place the eye at the extremity of a horizontal bar of iron or dark coloured wood, exposed to the sun's rays, and, in the direction of this bar, look at small objects at the distance of one or two hundred paces. For this surface, being heated by the rays of the sun, communicates its temperature to the strata of air in immediate contact with it, dilates them, and gives them an

elastic force sufficient to sustain the weight of the superior strata. Now we have seen that the ratio of refraction of the air depends upon its density simply; consequently the strata situated immediately over the bar, will refract the light less than those immediately above them; and these again will refract it less than those which follow, until by a progressive but rapid gradation, we arrive at strata whose distance from the bar removes them from the influence of its temperature; from this point the ratio of refraction will be sensibly constant. Hence it is clear, that if through such a medium we look horizontally at distant objects, situated in the direction of the bar and at a small height above it, we shall see two images, the one superior and upright, through the stratum of uniform density; the other inferior and inverted, through the strata of variable density. This curious observation, together with many others relating to the same subject, is due to Dr Wollaston.

85. The cause of these phenomena being once known, we can easily imitate them by methods of our own, which, besides the advantage of presenting them with certainty, have likewise that of exhibiting them in their full developement. A convenient apparatus for this purpose consists of an iron vessel of a rectangular shape, suspended about five feet from the ground by means of iron wires. In the direction of the sides of this vessel, at the distance of fifty or sixty feet, are placed small signals of paper of a triangular form, the vertices corresponding very nearly with the visual ray which grazes these sides. This done, we fill the vessel with coal partially ignited. Then, if we place the eye in the direction of the sides of the vessel, at the end opposite the signals, we shall see the image become double, as the temperature is raised, presenting distinctly two white points, one direct and the other inverted, which will approach each other in proportion as the visual rays come nearer grazing the sides, and at length join each other; and this reflection, produced by the excess of temperature of the air in contact with the heated iron, takes place, as well on the vertical sides of the vessel as on the horizontal bottom.

86. The same effect is sometimes very conspicuous in the strata of air contiguous to a dry and sandy soil, strongly heated by the sun. The density of the air, in this case, increases from the surface to a certain height, commonly very small, after which

it becomes for a short space sensibly uniform, and at length decreases very slowly, according to the common constitution of the atmosphere. If we suppose an observer placed in a stratum of mean density, and looking at a distant object situated also in this stratum, he will be able to see it in two ways; directly through the uniform stratum which intervenes, and indirectly by the rays reflected from the inferior stratum. These rays at first directed from the object towards the terrestrial surface, with a certain inclination, enter the strata of less density, are refracted there, taking a direction more nearly approaching to a horizontal line, whence they rise, and traversing the superior and denser strata which attract them, pass on to the eye of the observer. There will then be two images of the object, the one upright, by direct vision, the other inverted, by reflection. If the object be insulated on the dark ground of the sky, an inverted image of the sky will surround also the reflected image of the object exactly as when objects are represented by reflection on the surface of water.

87. We are hence able to explain a very curious phenomenon, known to French mariners under the name of *mirage*,† and which the French army had many opportunities of observing in their expedition to Egypt. The land of Lower Egypt is a vast horizontal plain, the uniformity of which is interrupted only by some small eminences which serve for the sites of villages, and which are thus rescued from the inundations of the Nile. Morning and evening, the appearance of the country agrees with the actual disposition and distance of the objects; but when the surface of the soil is heated by the presence of the sun, the ground seems to terminate at a certain distance by a general inundation, and the villages beyond resemble islands situated in the midst of a large lake. Beneath each village is seen its inverted image exactly as it appears in water. As we advance, the limits of this inundation appear to recede; the imaginary lake which seems to surround the village retires; and at length entirely vanishes, while the appearance is reproduced with respect to another village more distant. Thus, according to Monge, from whom this account is borrowed, every thing conspires to complete an illusion which is sometimes painful, especially in the

† The English word *looming* is sometimes used to denote this appearance, generally however it is employed in a more restricted sense.

desert, since it tantalizes one with the show of water at the very time when he has the greatest need of it.

A similar appearance is exhibited at sea in very calm weather. A ship seen at a distance and in the horizon, sometimes presents two images, one direct, the other inverted. The second answers exactly to the first, and is equally distinct; indeed the effect is precisely that of a reflection from a mirror. Hence the name, *mirage*, which mariners have given to this phenomenon. As it is produced by the difference of heat between the water and air, it is commonly seen after sudden changes of temperature; since the density of the water at the surface of the sea does not admit of its taking these variations so rapidly as the air. But on the other hand, the temperature of the water, and the evaporation which is constantly going on at its surface, prevent it from taking so high a temperature as the sandy surface of an arid soil. For these reasons, the phenomenon of double images occurs rarely at sea, and continues only for a short time; whereas in Egypt and on some sandy plains in which the same circumstances present themselves at the same height of the sun, it occurs every clear day.

88. M. Mathieu and myself observed at Dunkirk, on the seashore, many phenomena of this kind, and I have given the mathematical theory of them.* I have proved that the consecutive paths or trajectories, traced from the eye of the observer, interest in their second branches, in such a manner as to form a caustic, below which no point can be seen. The curve *LT* Fig. 72. represents this caustic, and *DMS* is the limiting trajectory, drawn from the eye of the observer, a tangent to the ground. I call it limiting trajectory, because it limits the height where the inversion takes place. In this figure all the points situated above the trajectory can send only one image to the eye of the observer. Those situated in the space *SLT* send two, the superior being erect, and the inferior inverted. Lastly, those situated below the caustic in the space *MLT*, not being capable of sending any, are invisible; so that a moving object, a man, for example, withdrawing successively to different distances, will present the successive appearances represented in figure 73.

Theory and experiment agree in proving that no very considerable difference of temperature is necessary in order that these

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* Memoirs of the Institute for 1809.

appearances may be produced. Two or three degrees of Fahrenheit's thermometer are sufficient, when the place of observation is a level and extended plain, which allows the luminous rays to be prolonged without obstacle, and thus to manifest the curvature of the trajectories described by them. Of this kind was the station selected at Dunkirk, it being a sandy beach situated in the downs, near fort Risban; and the observations were also favored by the existence of a great number of distant objects, such as steeples, trees, and cottages, which, rising like so many signals above the dry soil, manifested the course of the rays by the appearances they presented. Moreover, the phenomenon of the doubling and inverting of the images, presented itself almost every day, arising from differences of temperature not exceeding three degrees of Fahrenheit.

89. If we suppose a like difference of temperature to exist between two strata of air, not placed vertically one above the other, but laterally contiguous, the phenomenon will still present itself; only it will take place in a horizontal direction perpendicular to the common surface of the two strata. The late M. Jurine and M. Sorret observed an appearance of this kind on the lake of Geneva, at a place rendered very narrow by the approach of the opposite shores. Now upon examining the configuration of these shores, of which the one situated on the south side is bordered by high mountains, and the other, situated on the north side, is exposed to the rays of the sun, it will be easily perceived that such a difference of aspect, in calm weather, or even when a moderate wind is blowing parallel to the direction of the shores, may occasion for a moment, in the mass of air incumbent on the lake, a lateral inequality of temperature, and consequently of density; and this may produce inflexions in the luminous rays which traverse the mass of air in a longitudinal direction sufficiently near the limit where the lateral variation of density takes place.

90. It sometimes happens also, that distant objects seem suspended in the air; the image in this case is single, erect, and apparently not accompanied with any inverted figure.*

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* This phenomenon is called by the French *suspension*, and by the English *looming*. It has been shown in the memoir above cited, that when this phenomenon occurs, the second inverted image exists, but is infinitely reduced, so that we see only the erect image which is detached, on the inverted image of the sky.

When vision thus takes place by trajectories convex towards the earth or sea, the reflection is negative; the apparent horizon is much lower than it ought to be, for the height where the observation is made. Mariners ought, therefore, to be on their guard with respect to this phenomenon which tends to occasion considerable errors in latitude. It is found, indeed, by experiment, that these errors often amount to four or five minutes. The apparent horizon will therefore be too low when the sea is warmer than the air. If, on the contrary, its temperature is lower, its density is increased, but by a much more rapid law than ordinary; and the apparent horizon is sometimes raised very considerably. These errors may be avoided by observing the height of the stars, not above the horizon of the sea, but above an artificial horizon, placed without the inferior strata, where the extraordinary variation of density always takes place. But this expedient is not always easy even on land; and on board of vessels it is altogether impracticable, on account of the motion of the sea. In this case the error may be corrected, by taking, if it be possible, the distance of the two opposite horizons of the sea. The excess of the sum above two right angles, will give double the apparent depression of the horizon which it is necessary to employ in the calculation. Thus we shall know this depression by taking half the result. Unfortunately this observation of the two horizons, proposed by Dr Wollaston, appears very difficult to be taken with accuracy; and besides, we can seldom expect to find the differences of temperature between the air and water to agree sufficiently to render the two depressions equal. If it is not in our power to correct the error to which we are liable under these circumstances, it is at least well to be apprized of its existence and tendency, in order that we may not be misled by it.

Double Refraction.

91. It has already been stated, that a beam of light in traversing crystallized bodies, is generally divided into two portions of which the one, called the *ordinary ray*, follows the ordinary law of refraction assigned by Descartes; while the other, called the *extraordinary ray*, is subject to very different laws.

According to a very ingenious remark, first made by Dufay and confirmed by all succeeding observations, this phenomenon takes place in all crystals, except those whose primitive form is the cube or some of its geometrical derivatives. The separation of the refracted rays, proceeding from an incident beam, is more or less considerable, according to the nature of the crystal, and the direction in which the light traverses it; but this last variation is subject to the same law for all substances. In general, there are two directions, and only two, in which the separation of the two rays is infinitely small or nothing, and their velocity the same. These two directions are called the *axes of the crystal.* In proportion as the refracted rays deviate from the axes, their velocities of transmission become different; and this diversity is manifested, in general, by the separation or divergence which takes place in the rays, when the obliquity of incidence or emergence is the same. The inequality of the velocities increases according as the refracted rays form greater angles with the two axes; and it is the greatest possible when these angles are both right angles, that is, when the refracted rays become perpendicular to the plane of the two axes. Among the different kinds of crystals there are some in which the angle formed by the two axes is absolutely nothing; these may be regarded therefore as having but one axis. It is thus indeed that we did at first consider them; but it is better to imagine two axes, coinciding with each other, in order to preserve the analogy, and for the purpose of comprehending all crystals under one law. This case, however, is evidently more simple than the general one, because the inequality of the velocities depends entirely upon a single angle; on this account it will be examined first; and to make the process still more easy, we shall take, for an example, the rhomboidal carbonate of lime, commonly called Iceland spar; which, besides being remarkable for a double refraction extremely energetic, has likewise the advantage of being frequently met with in collections of minerals.

92. The crystals of this substance present cleavages in three directions, remarkable for their evenness and the facility with which they are effected. They are hence often separated of themselves into an infinity of small rhomboidal solids, having each six faces, parallel two and two, and formed of similar par-

Double Refraction

[illegible]

glass DG shall be nearly parallel. The junction of the two prisms is effected by heating them and melting between their surfaces some small grains of very pure mastic in tears, which are pressed out into a very thin transparent lamina. This lamina after cooling is sufficient to make the two surfaces adhere strongly and to determine the passage of light from one to the other, so that vision becomes possible through the double prism. Then we place this prism on the superior base of the column Hh, applying it by its face BC, to the column, which requires that the face CF of the glass should extend a little beyond BC, or should be in its prolongation; and in order that the crystal may remain fixed before the division AZ, in the situation judged most convenient for the experiment, we first put on the edge of the glass a few small drops of inspissated spirits of turpentine, which are sufficient to make it adhere. This being done, we place the eye at V, behind the posterior face of the glass, and look through the double prism at the vertical division AZ. In virtue of the two refractions which the crystal causes each ray, emerging from it, to undergo, it appears double; so that, properly speaking, we see two scales of division, placed one above the other, in a common direction, at least while the lateral deviation is nothing, as we now suppose it. But the inequality of the two refractions, added to that of the distance, causes the homologous marks of the two refractions to separate unequally from each other. If in certain parts the deviation is half an interval between the marks, a little further it is a whole interval; and at this place the two divisions will coincide, the coinciding marks, however, not being homologous. Further on, the coincidence ceases, and the marks of the two divisions again separate; but at some distance from this, their deviation being increased to an entire interval, the two divisions again coincide. If, for example, the division marked 451 coincided, in the first case, with the division marked 450, so as to give a deviation equal to one of the spaces; at the second coincidence the deviation would be two spaces; that is, if one the divisions was 502, the other would be 500. It would be three spaces if the coinciding divisions differed from each other by three units, and so on. Now, to follow out the consequences, let us take one of these cases, the second, for example. Since the marks 502 and 500 coincide, when seen through the double prism; this proves that the ray, emanating from the mark 502,

place the eye in this plane, and turn the crystal gradually upon its base, until the two images of the right line AC, take a common direction; and present a partial superposition, as in fig. 76. Then, as we know that the ordinary image always remains in the common plane of incidence and emergence, the extraordinary image which coincides with it will be found there, in like manner. This coincidence is observed when the straight line AC, the object of view, bisects one of the obtuse plane angles of the rhomboid, or is parallel to such bisecting line. Then the divergence of the two images, perpendicular to the plane of incidence, becomes nothing; and consequently, whatever be the forces which produce the extraordinary refraction, it is certain that their resultant is comprehended wholly in this plane. On this account it has received a particular denomination, namely, *the principal section of the rhomboid.* If we suppose the crystal on which these experiments are made to have the precise primitive form which belongs to carbonate of lime, the bases of the rhomboid will be perfect rhombuses; and then the principal section will contain the small diagonals drawn upon each base through the vertices of the obtuse angles. The plane of this section will then cut the rhomboid so as to form a parallelogram $ABA'B'$, in which the sides AB, $A'B'$, are the small diagonals Fig. 77. of the opposite rhombuses, and AB', $A'B$, the edges which connect them in the rhomboid. The line AA', drawn through the two solid obtuse angles A, A', is called the *axis of the crystal*; it is equally inclined to all the faces, and forms with them an angle $A'AB$ of 45° 23′ 25″, the angle $B'AA'$ being 63° 44′ 45″. To this we are to refer all the phenomena of extraordinary refraction, if we would obtain for them symmetrical and simple laws. Now this line exists only in crystals of Iceland spar, whose form presents outwardly the relations of symmetry which belong to the primitive particle. It is likewise necessary to conceive its existence in each small element of all these crystals, whatever be the exterior configuration which art or nature has given to the sensible mass resulting from their aggregation, and it forms there the same angle among its faces. If, in one of these small elementary rhomboids, we imagine a plane normal to one of the faces and dividing its obtuse angle into two equal parts, this plane will be parallel to all the similar planes which can be drawn through each of the other ele-

described; in the first place, the precision of the measurement extends to the divergence itself of the two rays, and not simply to the absolute incidence, in which a small error is seldom of any importance; in the second place, the alternations of coincidence and separation of the marks perform, in a manner, the office of verniers, by enabling us to fix with extreme accuracy the point where each coincidence is most perfect. The precision is rendered still greater by the facility with which we can move the column Hh to a greater or less distance from the vertical division which enables us to vary the extent of the coincidences, and thus to fix the precise distance at which each takes place most exactly.

108. According to what has been already observed, the course of the ray which experiences one of the two refractions, is the element from which we set out in calculating the course of the ray which experiences the other refraction. The first determination is easy in crystals with one axis, where one of the refractions always follows the law of Descartes; and in this case, we set out from the ordinary refraction, as being most simple. It is then necessary to determine the constant ratio which is its characteristic. This can be done simply by the above apparatus. For this purpose, it is sufficient to cut in the crystal prism a face so little inclined to the others, as to admit of our observing the
Fig. 173. refraction directly through the angle thus formed. Then we put this angle on the column Hh by one of its faces, and place the eye behind the other, at V, for example, and observe by refraction the ordinary image of O, one of the points of division, and note at the same time to what point R the prolongation of the emergent ray $I'V$, which comes to the eye, corresponds. The positions O, R, being thus known, with the height of the point of incidence and the refracting angle of the prism, we can calculate the ratio of refraction.* It is only necessary to be well assured that the refraction which we observe is of the ordinary kind. As to this we may satisfy ourselves either by verifying the constancy of its ratio under different incidences, and with every variety of direction as to the cutting; or by observing the position which the ray thus refracted takes with respect to the other, in virtue of the law of extraordinary refraction,

* For the formulas see Biot's *Traité de Physique*, tom. iii, p. 209.

supposed to be known; or lastly, by means of a modification acquired by the luminous rays in the act of double refraction, which Malus, to whom we are indebted for the discovery, has called the *polarisation of light.* This modification, being different for the two rays, will enable us to distinguish them; and for this purpose it is sufficient, as we shall show hereafter, to observe the refracted division through a thin plate of tourmaline. The preceding method of observing refraction admits of a dispersion; but if the prism be not very open, and if we do not place it very far from the division which we observe, this will not prevent the refracted marks from being still distinguishable with sufficient clearness. Finally, we avoid these inconveniences by placing before the eye a coloured glass, which allows only one kind of light to pass, as red, for instance. It is necessary, however, in this case, to reduce the results by calculation to what they would have been if we had observed the refraction of the mean rays.

109. In crystals of two axes, where the two velocities are variable, there are also some modes of cutting, in which one of the velocities is found to have a constant value for all directions comprehended within it; so that they offer the same facility as crystals of one axis, for determining the constants of double refraction. Suppose, for example, in the plane of the two axes, the two straight lines at right angles to each other, of which the one is a middle or intermediate line between the axes, the other, called the supplementary line, dividing the supplement of their mutual inclination into two equal parts. If we form in the crystal two plane sections at right angles to each other, of which the one is perpendicular to the middle line, and the other to the supplementary line; one of the velocities will be constant in the first section, and the other in the second; so that we shall have in one or the other of these directions a constant ratio of the sines. These remarkable results were first made known by M. Fresnel, as the consequences which he had deduced from his speculations respecting the nature of light.

110. But there is another and a direct method by which we can verify this constancy of the velocities in certain sections of a crystal or other substance. We cut an artificial face perpendicular to each of these sections; we then cause a ray to be reflected inward with different incidences, and see if the angle

of reflection is found to be equal to the angle of incidence, under all inclinations; for this equality is a consequence of the constancy of the velocity and is a characteristic by which it is known. To determine, therefore, whether it takes place, we have only to adapt to a parallelopiped of the substance under examination, another parallelopiped formed of glass or any other substance producing simple refraction which has been wrought at the same time with the first, in such a manner as to have all its faces common, and compare at one view the directions of the images of a small distant object, as the flame of a taper, for instance, reflected inward at the second surface of both bodies. For if, as we have supposed, the distance of the observed object be very great, compared with the dimensions of the double parallelopiped, the two reflected images must present equal deviations when the velocity of the reflected ray is constant in each of the two substances; and a small difference between these velocities, like that which is capable of being produced by substances hitherto known, will have absolutely no sensible effect in disturbing this equality, as may be easily ascertained by calculation. It is sufficient then to observe whether this equality exists, and continues through the progressive different inclinations which may be given to the reflected and incident rays by moving the double parallelopiped before the eye.

111. To avail ourselves of the operations performed by the preceding methods, as well as by other similar ones, it is necessary that the position of the axes of the substance on which we operate, should be determined with exactness in each of the fragments observed. This is easily done when the substance has been already examined, and the direction of its axes indicated with reference to some one of its natural faces; for then it is sufficient to separate a fragment at the natural cleavage, which will make known either this face itself, or some other related to it according to the laws of crystallization; so that we shall know the position of the axes in the fragment thus obtained, and consequently in the rest of the mass from which it is taken. When the substance which we observe is new, or when the position of its axes is not yet known, the fixing of these lines by double refraction alone, might require many ineffectual trials, especially on account of the aberration caused by the lateral deviations; but the polarisation of doubly refracted rays, offers for this purpose

a simple and sure indication by the aid of which the axes may be directly discovered with ease and accuracy. We may suppose, therefore, that their direction has been determined by this or some other process, and that it is actually known in the crystals which we wish to subject to observation.

This being done, suppose that by means of the apparatus above described, or some other similar method, we have determined for a certain crystal, the divergence of the rays for different directions about its axes; it remains to ascertain the progression of these divergencies, and form their general expression in relation to the axes of the crystal. Huygens, as before stated, has done this for Iceland spar, by means of a remarkable law which he connected with the theory of the undulations of light; but this same law has since been deduced by Laplace, from the theory of its materiality.

If we regard light as a material substance emitted from the luminous body, the refraction of the rays which traverse transparent bodies, is produced by the attractive forces, which the particles of these bodies exert upon the luminous particles, forces whose effect is sensible only at very small distances, and which in this respect are precisely similar to those which belong to chemical affinity. Accordingly, when a ray of light penetrates a refracting surface obliquely, the curved portion of the path which it describes, has only an infinitely small extent, inappreciable by our senses; so that the ray appears to be suddenly broken and changed, as to its direction, at the point of incidence. Now since the curve which it describes is imperceptible, we cannot deduce from its form, the nature of the forces which act upon the luminous particles at each point, as we discover the law of universal gravitation, from the form of the orbits described by the planets and comets; and we are accordingly reduced to the alternative of conjecturing the nature of these forces, from indirect inductions, and then of verifying it by the agreement of the results with experiment. Newton has done this for ordinary refraction, or where the velocity is constant, by considering each luminous particle which traverses a refracting surface, as acted upon before, during, and after, its passage, by attractive forces, sensible only at very small distances, and emanating from all the particles of the refracting medium. This definition specifies nothing respecting the law according to

which these forces diminish in the space through which they are sensibly variable; it allows us only to calculate the resultant for each distance, and to suppose them constant when the distance becomes sensible. Now these data are sufficient to enable us to calculate, not the variable velocity of the luminous particles in their curvilinear motion, nor the nature of this motion, but only the ratio which exists between the velocities and the definite directions which they take, either within or without the refracting medium, when the distance of the luminous particles from the refracting surface has become so great, that the course of the ray is sensibly rectilinear; and this comprehends all the limits of distance within which we are able to observe.

112. In the case of extraordinary refraction, produced by crystallized bodies, we have not even this advantage of being able to define the origin of the molecular or elementary force, nor the manner in which it emanates from each individual particle of the crystal; for our representation of the phenomena by attractive and repulsive forces emanating from the axes, is only the indication of a combined result, and not the expression of a molecular action. All that we know, therefore, in this case, or at least all that we can suppose, when we adopt the hypothesis of the materiality of light, is that the forces, whatever they may be, which act upon the luminous rays, in this as in every other case, emanate from the particles of refracting bodies, and are attractive or repulsive; whether they exert a power of the same kind upon all the luminous particles, or whether the nature of this power varies according to the peculiar modifications of these particles. Now in cases where a material particle is acted upon by such forces, its motion is subject to a general mechanical condition, called *the principle of least action.* By employing this together with the particular condition, that the forces are sensible only at very small distances, Laplace has obtained two equations, which determine completely and generally the direction of the refracted ray, for each given direction of incidence, when we know the law of the definitive velocity of the luminous particles, in the interior of the refracting medium, at a sensible distance from its surfaces.

113. In the case of ordinary refraction, the definitive velocity is constant; for the deviation of the ordinary ray is the same in the same body, in whatever direction it be tried, provided

the incidence is the same, and the surrounding medium does not change. Also when we suppose the interior velocity constant, the equations deduced from the principle of least action, show that the refraction takes place in the prolongation of the plane of incidence itself, and according to the constant ratio of the sines, which is in fact the physical law of ordinary refraction in all bodies hitherto observed.

Hence it was natural to suppose that extraordinary refraction was produced by a velocity varying according to the direction of the ray, about the axes of the crystal. Beginning, therefore, with crystals of one axis, we have seen that double refraction is exerted symmetrically about the axis; and that, being nothing in the direction of the axis itself, it attains its maximum when the rays cut it at right angles. For the crystals in question then, we must confine ourselves to the laws of velocity which satisfy these conditions. Laplace has tried the following equation,

$$v'^2 = v^2 + k \sin^2 U,$$

in which v represents the ordinary velocity, v' the extraordinary, U the angle formed with the axis by the extraordinary ray, and k a constant coefficient for each crystal. The substitution of this law of velocity in the equations derived from the principle of least action, gave him at once the law of Huygens. This law was at first completely verified only for Iceland spar; I have verified it also for rock crystal and several other substances. Generally the coefficient k, is positive in crystals of double refraction attractive, like rock crystal, and negative in the others. Its absolute value changes, likewise, with the substance. Sometimes, indeed, it has variations for different specimens of the same kind of mineral, which may be owing to slight differences of composition or structure; but amidst these different modifications, the same law of velocities applies to all crystals of one axis, hitherto observed.

114. Now for crystals of two axes, the extraordinary velocity v' must be supposed to depend upon the two angles U, U', which the axes form with the refracted ray. Analogy, therefore, leads us to try whether the square of this velocity may not also be expressed by a function of the second order, but more general, that is, having relation to these two angles. Now, since

in every crystal of two axes, the two refractions become equal when the refracted ray takes the direction of one or the other of the axes, it follows, that in this case, the extraordinary velocity becomes equal to the ordinary. This condition limits the generality of the function of the second order, and reduces it to the following form;

$$v'^2 = v^2 + k \sin U \sin U';$$

that is, there remains only the product of the two sines.*

By introducing these formulas into the equations derived from the principle of least action, we determine the course of the rays for all cases, and it only remains to see whether this course agrees with the results of experiment. In order to make the comparison in a rigorous manner, I have instituted, by the method of coincidences, and with the aid of the apparatus above described, a great number of experiments upon the white topaz of Saxony, the yellow topaz of Brazil, the anhydrous sulphate of lime and euclase, which are crystals of two axes, and of a form and composition altogether different the one from the other. The measures given by experiment are found to agree with the formulas most perfectly. I have observed a similar agreement in several other crystals, by applying the same law to a different class of phenomena, but yet connected with double refraction. This accumulation of proof leaves no doubt, that the law, indicated also by such strong analogies, is, in fact, the law of nature. It may be remarked, that the law under consideration comprehends, as a particular case, the law of Huygens, for crystals of one axis; since, by considering these crystals as having two axes united in one, we have their mutual inclination equal to zero. Then the two angles U, U', formed with the two axes by the refracted ray, become equal; and the increase of the square of the velocity contains only the square of the sine. In this investigation I have supposed one of the two velocities to be always constant in the same crystal, whatever be the direction of the transmitted ray. This is indeed the opinion generally entertained; and since, by adopting it, and apply-

* I presented this law to the Institute with the observation by which it is proved, the 29th of March, 1819.

ing to it the principle of least action, all the *phenomena of divergence* which I have observed, were found to be represented with an exactness which left no room for sensible error, even in crystals whose double refraction is the most energetic, I confined myself to some direct attempts to verify this constancy; and finding that they appeared to confirm it, I dismissed all doubt upon the subject. Meantime the experiments of M. Fresnel, which I have myself verified both by his method and by that of interior reflections, already explained, show that the supposition of a constant velocity is not exact. But by two singular circumstances it appears why, by adopting it, the observations are capable of being so faithfully represented. On the one hand, the variableness of the velocities, which increases with the angle formed by the axes, is found to be exceedingly feeble and almost insensible, in the most energetic crystals which have been made use of; and on the other, this variableness is such, that the difference of the square of the velocities preserves the form which I have assigned it; that is, for each couple of rays which have the same direction of interior transmission, it is, according to M. Fresnel, really proportional to the product of the sines of the angles comprehended between this direction and the two axes of the crystal; whence we see that, by reason of the feebleness of double refraction in all known substances, the divergence of the rays, which is sensibly proportional to the difference of the velocities, is found to be expressed as exactly by the false supposition of a constant velocity, as by the true supposition of its variableness.

115. The ideas of M. Fresnel concerning the nature of light, suggested to him a geometrical construction, by which he represented generally the velocities of rays refracted by any crystal whatever. This construction, represented algebraically, gives the following expression for the velocities of two rays which accompany each other in their passage through the crystal, that is, which traverse the crystal in the same direction. Let v, v', be the velocities of these two rays, and U, U', the angles which they make with the two axes of the crystal; and let n and k be two constants, belonging to the two substances. We shall have,

$$v^2 = n^2 + k \sin^2 \tfrac{1}{2}(U' - U),$$

$$v'^2 = n^2 + k \sin^2 \tfrac{1}{2}(U' + U).$$

Whence we obtain generally,

$$v'^2 - v^2 = k\left[(\sin^2 \tfrac{1}{2}(U' + U) - \sin^2 \tfrac{1}{2}(U' - U)\right],$$

or by developing the multiple sines of the second member, we shall have

$$v'^2 - v^2 = k \sin U \sin U';$$

that is, the difference of the squares of the velocities is proportional to the product of the sines of the angles formed by the rays with the two axes, agreeably to what we had before discovered by experiment. When the crystal has only one axis, the two angles U, U', become equal, and then $U' - U$ is constantly nothing. Hence it follows that the velocity v reduces itself to n, and is then constant in all directions. This velocity, which depends generally on the difference of the two angles U, U', answers then to what we call *ordinary;* and the other v', depending on their sum, is that which we call *extraordinary*. To abridge the enunciations, we shall hereafter adopt this mode of expression.

Although the velocity v is, generally speaking, variable in crystals of two axes, yet it becomes constant in these same crystals, when the refracted ray is comprehended in a plane situated intermediate between the axes and perpendicular to the supplementary line; and this is what the formula represents; for then $U = U'$, and the expression for v is reduced to the constant n. On the other hand, if we draw a plane through the supplementary line perpendicular to the mean line, all the rays comprehended in this section will form with the two axes, angles U, U', supplementary to each other, and consequently their sum will be always constant and equal to 180°. Thus in this section, the sine of the angle $\frac{1}{2}(U' + U)$ will have a constant value equal to unity; whence we see that the velocity v' will then become constant, and will have its square constantly equal to $n^2 + k$. These are precisely the first results at which M. Fresnel arrived.

116. If we represent by n' this particular value of the velocity v', k will be expressed by $n'^2 - n^2$, and the general expressions for the two velocities v, v', will be

$$v^2 = n^2 + (n'^2 - n^2) \sin^2 \tfrac{1}{2}(U' - U);$$

$$v'^2 = n^2 + (n'^2 - n^2) \sin^2 \tfrac{1}{2}(U' + U),$$

in which the two constants n, n', have now a physical signification.

We do not undertake to say generally that the law given by this philosopher, embraces all the phenomena, for we have not yet examined the subject sufficiently to hazard such an assertion; but we are at least convinced that it unites a great number, and that it satisfies particularly all those which relate to the divergence of rays simultaneously refracted. Accordingly we shall adopt it in what we have to say hereafter with respect to these phenomena.

117. It follows from the foregoing considerations, that in order to be able to predict the course of luminous rays, and their deviations for all possible directions, in a crystallized substance possessing the property of double refraction, it is sufficient to observe these deviations in a single fragment of this substance, in directions known with respect to the axes; and after comparing these deviations with the theoretical formulas, to deduce thence the values of the two constants n and n', of which one represents the ordinary velocity, and the other the extraordinary, when the angles U, U', are both right angles; that is, when the rays traverse the crystal perpendicularly to the plane which contains the two axes.

Here, as in simple refraction, the velocities v, v', are not rigorously the same for the different simple rays which compose white light. The images also, given by the two velocities, are generally coloured and dispersed as well as the ordinary images, though not in the same manner. We shall examine the laws of this phenomenon in detail, when we come to speak of the decomposition of light and the analysis of colours.

118. We shall also defer giving a particular description of a remarkable fact discovered by Mr J. F. W. Herschel, son of the celebrated astronomer; namely, that in the same crystal, the axes of double refraction relative to the different simple rays are often separated from each other, forming for each species of rays, couples unequally inclined, though always comprehended in the same plane and symmetrically disposed about the same straight line which bisects the angles of all the couples. This dispersion has been observed hitherto only in crystals of two axes. It hence results that with the same incidence, the angles U, U', formed by the refracted rays with the axes, are

different for different species of rays, which circumstance must affect the deviations they experience. Hence, in order to define rigorously all the phenomena, it is necessary to mention the particular species of rays with which they are supposed to be observed. Hereafter, when we use no limitation in this respect, it is to be understood as in the case of ordinary refraction, that regard is had to the mean rays of the spectrum, that is, to the green or yellow; or rather that we are speaking of a phenomenon in which the effect of dispersion is insensible or inconsiderable.

119. In general, for the same species of simple rays, the values of the coefficient k, or $n'^2 - n^2$, are different in crystals of different forms and different natures; whence it follows that double refraction exerted by these bodies is of very unequal energies. We find also that substances of nearly the same composition, act in this respect very differently. Common sulphate of lime, for example, has a very weak double refraction; whereas the anhydrous sulphate of lime has a very strong one. It is true that the primitive forms of these two substances are very different. But the pure carbonate of lime, and the magnesian carbonate of lime, both of which have the primitive form of rhomboids, and rhomboids whose angles are so nearly equal that the difference, though real and measurable, has been doubted, have also double refractions sensibly unequal. For the mean value of k, which is 0,543 for the first, according to my experiments, varies for the second from 0,581 to 0,591; and it is probable that this last variation is owing to a small difference of composition. Some beryls which do not differ externally, except in colour, give also values for k very sensibly unequal; and several specimens have even exhibited phenomena which seemed to indicate the existence of two axes. Hence we may conclude generally that when two crystallized substances differ in their composition or in their primitive form, they differ also in their property of double refraction; and reciprocally, that a difference as to double refraction supposes always a difference in composition or structure, which renders this kind of phenomena very useful in characterizing minerals.

120. The coefficient k, not only experiences variations in its value, in passing from one substance to another, but also in its sign. It is positive for some substances and negative for others.

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over, the preceding definition embraces the construction which we have assigned to this section. For, if we suppose the two axes to approach each other until they form one line, these three planes will become one, which will be drawn through the single axis and through the normal to the face we are considering, which is precisely the construction of the principal section for crystals of one axis, according to the definition we have given.

Reflection at the second Surface of Crystals.

122. The theory we have been explaining, is not confined to rays refracted extraordinarily by crystals. It applies also to those which are reflected inward at their second surface. But before entering into a detail of the consequences thus indicated, it is necessary to establish by experiment the principal characteristics of this kind of phenomena.

When a ray of light falls upon the first surface of a crystal, coming through a vacuum or any medium not crystallized, it is partially reflected in a single beam, in such a manner that the angle of reflection, reckoned from the normal, is equal to the angle of incidence. The attractive or repulsive force which emanates from the axes of the crystal, appears to have no influence on this phenomenon; for we may turn the crystal upon its plane in all possible directions without altering the intensity or the direction of the reflected ray. But it is not so with respect to the interior reflection which takes place at the second surface of the crystal. Each ray reflected at this surface, is generally divided into two portions which return into the crystal, the one experiencing the ordinary refraction, the other, the extraordinary, these terms being used in the sense already explained.

123. In order to comprehend the cause of this division, it must be understood that rays, refracted either ordinarily or extraordinarily, when they have penetrated into the interior of the crystal to a sensible depth, acquire such a mode of arrangement of their particles, that they can no longer be divided during their course through this crystal; and experiment proves that they would no longer be divided if they should traverse a second crystal contiguous to the first, and having its principal section

directed in the prolongation of the first. This particular mode of arrangement constitutes what Malus has termed the *polarisation of light.* Now when the particles which compose the same ray, refracted ordinarily or extraordinarily, approach the second surface of a crystal, at a distance sufficiently small to experience the influence of the reflecting forces proceeding from it, it happens, in general, that a certain number of particles are turned by these forces into directions different from those derived from refraction; so that in returning into the crystal by the effect of a total or partial reflection, they become anew susceptible of being divided into two refracted portions, ordinary and extraordinary. I say, in general, for there are certain particular positions, in which the reflecting forces do not alter the arrangement originally given by refraction to the luminous particles; and then the ray is reflected without being divided, or even escapes reflection entirely. It is sufficient for the present, to observe that the forces in question only influence the intensity of the reflected portion, and not the direction derived from reflection. A ray which is reflected single, or which emerges from the crystal without being reflected, would suffer double reflection, if the particles which compose it were otherwise disposed; as may be verified by experiment. Accordingly the direction of the reflection is the first thing to be determined.

124. This determination is easily made with respect to crystals of one axis, in which one of the velocities is constant. It is sufficient to rely on this fact, that the reflected ray must be affected on re-entering the crystal, as a ray would be, which came from without, and whose particles had not originally received any particular disposition. Now in case the crystal has only one axis, the direction of the return is determined by the known reflection of the ordinary ray having a constant velocity. Let I' be the point of Fig. 30
interior incidence, and $O'I'$ the incident ray. If it has suffered ordinary refraction, construct the ordinary reflected ray $I'O''$, which makes the angle of reflection equal to the angle of incidence, on the other side of the normal $I'N'$; then calculate by the theoretical formulas, the direction of the extraordinary ray $I'E''$, which corresponds to it at its departure from the point of reflection I', that is, which has proceeded from the same exterior incident ray; we shall thus have the two reflected rays which result from the division of the incident ray $O'I'$, after reflection.

263. These measurements Newton made. He took the diameter of the simple rings *of the same order*, at the interior and exterior part of their perimeters, considering them successively at the limits of the different colours of the spectrum, beginning with extreme violet. According to his constant practice, he took care to connect these results by a mathematical law, which should represent them with sufficient accuracy. He thus found that the diameters, whether interior or exterior, were to each other sensibly as the cube roots of the numbers $\frac{1}{2}$, $\frac{9}{16}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$, 1, which represent the lengths a musical cord must have, in order to produce all the notes in an octave; that is, if we represent by 1 the interior diameter of a certain ring, when it is formed by the red rays which compose the extreme part of the spectrum, $\sqrt[3]{\frac{8}{9}}$ will express the interior diameter of the same ring, when it is formed by the rays which are the limit between the red and the orange; and so on to $\sqrt[3]{\frac{1}{2}}$, which will express the exterior diameter of the same ring, when it is formed of the last violet rays taken at the other extremity of the spectrum.* Knowing the ratios of the diameters, Newton took the squares of these ratios, and they gave him the proportions of the thickness which the plate of air must have at the beginning and end of the observed rings. Similar measurements, made upon the different orders of rings formed by the same simple colour, showed that the intervals of the thicknesses where the reflection took place, were sensibly equal to those where transmission took place; so that if we designate generally by e_1, the thickness of the air at the commencement of the first bright ring formed by any of the simple rays, this ring ends with the thickness $3\,e_1$, and thus occupies an interval of thickness equal to $2\,e_1$; after which comes the first dark ring, occupying also the same interval of thickness $2\,e_1$; next to this in course,

* The numbers here employed by Newton, have to each other a very singular relation, which was first noticed by M. Blanc. It is this; if we multiply together two terms taken at equal distances from the extremes, for example, $\frac{9}{16}$ and $\frac{8}{9}$, $\frac{3}{5}$ and $\frac{5}{6}$, or $\frac{2}{3}$ and $\frac{3}{4}$, we shall always have a constant product $\frac{1}{2}$, which is equal to that of the two extreme terms. I shall hereafter make known the remarkable consequences which result from this relation.

total reflection begins in each crystal, depends upon its greater or less refracting power, and that of the exterior medium; but we cannot in the same way calculate its limits theoretically, because we are ignorant how the attractive or repulsive force, which proceeds from the axes of the crystal, varies near its surface. It is, therefore, necessary to recur to experiment, and to determine the commencement of total reflection by the impossibility of obtaining an emergent ray. I have given a detailed account of this calculation* for crystals of one axis, and have developed the remarkable consequences which result from it, in relation to the variations which the forces, proceeding from the axes, undergo near the exterior surface of crystals. Crystals of two axes offer similar considerations. One of the best means of ascertaining the nature of these forces, is to place the surfaces of the crystal in contact with transparent media of a greater refracting power than themselves, for instance, with phosphorus in a state of fusion, and to note the limits, as well as all the other particulars of interior reflection in media thus terminated. Perhaps we may be able to measure by this process, the double reflection of opaque bodies, as Dr Wollaston has determined their simple refraction. This investigation requires some theoretical principles which will be detailed in the following section.

Passage of Light through several Contiguous Bodies possessing the Power of Double Refraction.

126. We have supposed the preceding experiments to be made in a vacuum, or in the air, whose proper action upon light is so feeble as to have no sensible effect. We are now to inquire what takes place when the rays which penetrate a doubly refracting crystal, instead of entering from the air, pass out of a medium possessing the power of double or simple refraction.

Let us begin with this latter case, which is less complicated, and for still greater simplicity, let us suppose also that the crystal, into which the ray is to pass, has only one axis, in which

* *Traité de Physique.*

TABLE

Representing the thicknesses of the air at which the different rings begin and end, expressed in ten millionths of an inch.

General expression of the thicknesses e_n, E_n, at which each ring, of the order n, begins and ends.	Extreme violet.	Limit of the violet and indigo.	Limit of the indigo and blue.	Limit of the blue and green.	Limit of the green and yellow.	Limit of the yellow and orange.	Limit of the orange and red.	Extreme red.
1st ring e_1	1,99849	2,16154	2,25671	2,42071	2,61866	2,80899	2,93207	3,172206
$E_1 = 3\,e_1$	5,99547	6,48462	6,77013	7,26213	7,85598	8,42697	8,79621	9,516618
2d ring $e_2 = 5\,e_1$	9,99245	10,80770	11,28355	12,10355	13,09330	14,04495	14,66035	15,861030
$E_2 = 7\,e_1$	13,98943	15,13078	15,79697	16,94497	18,33062	19,66293	20,52449	22,205442
3d ring $e_3 = 9\,e_1$	17,98641	19,45386	20,31039	21,78639	23,56794	25,28091	26,38863	28,549854
$E_3 = 11\,e_1$	21,98339	23,77694	24,82381	26,62781	28,80526	30,89889	32,25277	34,894266
4th ring $e_4 = 13\,e_1$	25,98037	28,10002	29,33723	31,46923	34,04258	36,51687	38,11691	41,238678
$E_4 = 15\,e_1$	29,97735	32,42310	33,85065	36,31065	39,27990	42,13485	43,98105	47,583090
5th ring $e_5 = 17\,e_1$	32,97433	36,74618	38,36407	41,15207	44,51722	47,75283	49,84519	53,927502
$E_5 = 19\,e_1$	37,97131	41,06926	42,87749	45,99349	49,75454	53,37081	55,70923	60,271914
6th ring $e_6 = 21\,e_1$	41,96829	45,39234	47,39091	50,83491	54,99186	58,98879	61,57347	66,616326
$E_6 = 23\,e_1$	45,96527	49,71542	51,90433	55,67633	60,22918	64,60677	67,43761	72,960738
7th ring $e_7 = 25\,e_1$	49,96225	54,03850	56,41775	60,51775	65,46650	70,22475	73,30175	79,305150
$E_7 = 27\,e_1$	53,95923	58,36158	60,93117	65,35917	70,70382	75,84273	79,16589	85,649562

tal, still remains single; but it changes its refraction, from ordinary in the first, to extraordinary in the second, and *vice versâ*. Between these two limits of position, each ray, whether ordinary or extraordinary, coming from the first crystal, is divided into two when it enters the second, and these portions obey the laws indicated by the preceding constructions. But the intensity of each portion depends still upon the angle made by the two principal sections, increasing or diminishing with this angle, according as the motion of the principal sections causes the portion to diverge from or approach to the limit where it ought to disappear. Hence we must conclude, that the formation or non-formation of two portions in the second crystal, depends upon the physical modifications which the particles may have acquired in the first crystal, modifications which render them better fitted to undergo the one or the other refraction in the second, according to the directions of their faces, with respect to its axes; which does not prevent the theory from indicating with exactness, the *directions of translation*, which these particles would take, if their physical state permitted them to be shared between the two refractions. Similar cases have already been presented by refraction at the second surface of crystals; for, in fact, a ray reflected inward at the second surface of a crystal, experiences the same influence, as if it emerged entirely from the crystal to enter another or return into the same.

128. Hitherto we have supposed that the contiguous media were composed of crystals of only one axis. This supposition, by rendering one of the velocities constant, allowed us to calculate its propagation by Descartes' law of the sines being extended to contiguous media of different refracting powers. This expedient cannot be employed when one of the contiguous media is a crystal of two axes; since then its two velocities are generally variable; but we can supply the defect by considering that the rays, at the moment when they traverse the surface of contact, are without the sphere of action of the forces by which the double refraction is produced; so that they are really in the same condition they would be subjected to, if they had passed with their preceding velocity, acquired from an uncrystallized body, into another likewise uncrystallized, but of a different refraction. In this case, their definitive course would be still the same, if, instead of introducing them thus directly into the second

it emits from being mixed with foreign light. In this case, the lower border of the image will invariably appear painted red, and the upper one blue and violet.

Fig. 102. If we observe in this manner a very thin white object, for example, a white pin, a silver wire, a thread of white silk, or a narrow strip of white paper, placed on a dark ground, parallel to the edges of a prism, of a sufficiently refracting power, we shall perceive no white at all in the image *s s'*; but this image will appear entirely made up of parallel zones of different colours, among which we distinguish particularly, three distinct hues, the red at the bottom, the green in the middle, and the blue at the top.* Whatever be the nature of the substance whose image is thus observed, provided the colour be white, it exhibits, when seen through the prism, exactly the same series of colours; and if the dimensions of the several substances thus seen, are equal, it is absolutely impossible to distinguish one from the other.

144. Let us now endeavour to analyze this phenomenon and see the consequences to which it leads. The first circumstance we have to remark, is the dilatation of the image in the direction of its height. Indeed, if the object *SS'* were a mathematical line, parallel to the edges of the prism, and if all the rays which emanate from it were refracted according to the same ratio of the sine of refraction to the sine of incidence, the figure of the refracted image *s s'*, must also be a straight line without breadth; and though we cannot rigorously reduce an object to this limit, it is yet easy to see that, when the object is very narrow, as a pin, any further reduction is idle, since it does not sensibly change the size of the observed image. Moreover,

Fig. 101. whatever be the length *SS'* of the object, if all the rays coming from it are refracted through the prism according to the same ratio of the sines, it is evident from calculation, that we can always find such a position, that the angle *ROR'*, contained be-

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* To perform this experiment well, we should make use of a prism of flint glass, of a refracting angle equal at least to 60°. Then the zones are perfectly obvious and distinct. If we have not such a prism, we can construct one with two glass plates, inclined to each other at an angle of about 60° or more, the space between them being filled with pure water, or water saturated with the acetate of lead, in order to increase its refracting power.

tween the two emergent rays coming from its extremities, will be precisely equal to the angle SKS', contained between the two incident rays from which they are derived, that is, equal to the apparent diameter itself; at least if we suppose the object so far distant, that the distances $R'K$ and RO, of the point K and the observer from the prism, may be regarded as nothing compared with SI and $s\,I'$. The position of which we speak, is that in which the angle SIA is equal to the angle $OR'B$. Now this equality is far from what we actually observe; for whatever position we give to the prism, and whatever inclination it has with respect to the incident rays, we can never render the refracted image equal in extent to the direct image; and the difference is more sensible, according as the object is smaller. This result, which is confirmed and rendered general by all the experiments which follow, obliges us to conclude, *that the luminous rays which come from terrestrial objects, do not all observe, on being refracted, the same ratio of the sine of incidence to the sine of refraction.*

146. We have, moreover, remarked that the image of the pin, silk thread, &c., is composed of parallel portions of different colours. Hence, we must infer that the refraction is different for the rays which produce the sensation of the different colours; so that the dilatation of the image arises from the different refrangibility of the rays which cause these colours.

Again, since the light of the pin which produces all these colours, appears white when the eye receives it directly, before it is separated by the prism, it is manifest that what we call whiteness, is merely the union of a certain number of rays, which, considered separately, produce the sensation of the different colours, but which, united, produce the sensation of white. This may be easily verified. We have only to turn the pin, or strip of paper, in such a manner that its direction shall be perpendicular to the edges of the refracting prism, instead of being parallel to them. Then the upper extremity of the image is violet and the other extremity red. But if the object be of the same size throughout its length, these are the only portions of the image which appear coloured, and all the intermediate part is white, as if seen directly. Now it is manifest, that in turning the pin or strip of paper on itself, we do not change the manner in which light emanates from it. The rays, therefore, which proceed from each point, still undergo, in the prism, the same

267. We are thus enabled, by means of this figure, to explain in all its details, the phenomenon of coloured rings formed by natural light, when it is reflected by a thin lamina of air or any other substance; for all the simple rays which compose this light, entering the thin lamina together with an equal incidence, it follows that each must form rings there according to its own proper laws; and the second surface of the lamina must reflect or transmit them at the same thicknesses at which it would have reflected or transmitted them, if the simple rays had traversed it separately or successively. Since then, the same thickness may reflect separately the rays of different species, as we have seen, it may also reflect them simultaneously, while it allows all the others to pass. Hence it follows that if the lamina is throughout of the same thickness, it will reflect in all its points the same mixture of rays, under each given obliquity; and consequently, if we are at a sufficient distance to cause the rays which it sends to the eye from its different points to be sensibly parallel, it will appear of a uniform colour. But if the thickness varies, the colours will also vary in the different points of the lamina, according to the thickness. Thus, when the lamina is comprehended between two spherical object-glasses, between which the space constantly increases in thickness in every direction as we depart from the point of contact, there must be formed about this point an infinite number of circular rings of different shades, as we in fact observe in the coloured rings formed by daylight.

268. We can even determine the order in which the colours of these rings must succeed each other as we depart from their common centre. For this purpose, it is sufficient to conceive a straight line, which, being at first coincident with *CZR*, gradually removes from it, remaining always parallel to it, and passing in this manner through all the alternate spaces where reflection and transmission take place. When this line first leaves *CZR*, it traverses a space in which little or no reflection is produced, on account of the extreme thinness of the lamina; after which it arrives at 1, that is, at the feeblest beginnings of extreme violet. But as soon as it has traversed, ever so little, the space which belongs to this colour, it meets also those belonging to the blue and green, which, with the violet will compose a blue; then it will enter the yellow and red, which, with this blue, will compose a white. From the slight inclination of the line 11′ to the

axis CZR, it is evident that this passage from violet to white must be very rapid. In a lamina of air, for example, the beginning of the violet in the first ring answers to the thickness 1,99849, as shown by the table, article 264; and the commencement of the red in this same ring, or the limit of the red and the orange, answers to the thickness 2,93207; whence it follows that the separation of colours does not take place except at an interval of thickness equal to 2,93207 — 1,99849 or 0,99358; and consequently, it must be very difficult to distinguish it unless the thicknesses vary with extreme slowness. The white being once formed in the first ring, it will continue to be reflected more or less perfectly, while the moving line passes from 1 to 3; after which, the colours which compose it successively disappearing, it will change first into a compound yellow, then into a red, and this red will finally disappear at 3′. Here the colours of the second ring begin; and between these and those of the first ring there is a small dark interval, at least if we adopt as rigorously exact the limits of reflection fixed by Newton. For the extreme thickness where the red of the first ring *ends*, is 9,516618, according to our table; and the thickness where the violet of the second ring *begins* is 9,99245; whence it follows that between these two limits, there is an interval equal to 9,99245 — 9,516618, or 0,475832, in which no colour is reflected; consequently there must be formed by transmission in this place, a very narrow white ring. But the existence of these two rings may be modified by the more or less considerable extension of the sensible limits of reflection; for if the thin plate of air, upon which the preceding limits were taken, could suddenly exert a greater reflecting power, without its other properties being changed, we might observe sensible quantities of light, at the places where before we did not perceive any; and the intervals of thickness occupied by the bright reflected rings becoming more enlarged, the two first rings might extend so far, as to cause the narrow dark ring which separated them, in the determinations of Newton, to disappear. Indeed, we shall find substances hereafter, which present this superposition in a manner sufficiently distinct to be observed, not in this self-same phenomenon of the rings, but in a series of facts which are governed by precisely the same periodical laws, on a scale of thicknesses much more extensive. Returning, however, to the experiments of Newton, we see that beyond the dark narrow ring in

question, the colours of the second ring commence and succeed each other in order, while the moving line passes from 5 to 7; these are more vivid than those in the first ring, because they are more dilated and separated from each other, as the figure itself indicates, by the greater inclination of the line 77′ to the axis *CZR*. In consequence of this separation, there no longer intervenes a white between the blue and yellow of this ring, as in the first, but a mixture of orange, yellow, green, blue, and indigo, all which colours, joined together, must compose a faint imperfect green. In like manner, the colours of the third ring succeed each other in order; first comes the violet, which mixes a little with the red of the second ring; for it begins at the thickness 17,98641, and ends at 21,98339, whereas the red of the second ring ends only at the thickness 22,205442; whence it follows that these two colours are reflected together through the whole of the violet of the preceding ring. We hence see why this violet is not separately perceived, but instead of it a reddish purple. Next come the blue and green, which are less mixed with other colours, and for this reason are more lively than in the former case, especially the green. Then follows the yellow, of which the part next the green is distinct and good; but the other part, next the red, which comes immediately after, consists of a yellow which, as well as the red, is mixed with the violet and blue of the fourth ring; whence result different degrees of red inclining strongly to purple. This violet and blue, which should succeed this red, are mixed and confounded with it, so that in place of them there succeeds a green. This green at first inclines to blue, but it very soon becomes a fine green; and it is the only unmixed and lively colour which appears in this fourth ring; for in proportion as it inclines to yellow, it begins to mix with the colours of the fifth ring, by which mixture the yellow and red immediately succeeding, become very faint and dirty, especially the yellow, which, being the faintest of the colours, can hardly be perceived. After this, the different rings and colours are intermixed and confounded more and more, until after three or four successions in which the red and blue especially predominate, all the species of colours become mixed together in nearly equal degrees, and constitute an uniform white.

269. As we have observed that the rays of one colour are transmitted at the same place at which the rays of another colour

. parallel to each other, are arranged
efracting angle is a right angle, and
l, *A*, *A*, *A*. These prisms having each **Fig. 100'.**
placed upon the same plane, and being
their longitudinal edges, we apply to
of pasteboard, so as to cover them, and
d then one of steel; after which, we
a sort of vice, the jaws of which, acting
s of steel, press them against the oppo-
and consequently exert on these prisms,
axes a nearly uniform longitudinal pres-
, we introduce between these prisms thus
prisms *B*, *B*, *B*, *B*, of the same refracting
ind of glass, but in their natural state of
cement them to the first by a stratum of
ened by heating, which, by establishing the
rfaces, renders possible the passage of light
of eight prisms, notwithstanding the largeness
ngle. Thus, when a very slender luminous
rpendicularly through the first surface of this
s freely and nearly perpendicularly through
ver (which is the main point to be attended
d into two portions, whose sensible diver-
he same distance, according as the exterior
re considerable. In some cases, the diver-
6 of an inch for 40 inches distance, so that
mistake it; nor can we suppose that it is pro-
; of the path or trajectory, analogous to that
kes place in media of variable density; for,
he number of prisms, there would in this case
an two images. But the circumstance which
on the subject, is, that the two emergent por-
r in their absolute properties or in their rela-
the physical modifications which characterize
merging from a doubly refracting crystal.
iment of M. Fresnel, therefore, completely
or pressure imparts to glass the property of
but it does not teach us in what manner the
this property; nor can we conclude from it,
fraction of bodies naturally crystallized, is

lines, will be also equal to that of the first.* Consequently, the inferior extremity R', of the second image will be removed as far from the inferior extremity R of the first, as this last is from the direct image S, by the first refraction. And, as the same may be said of all the other points of the first image VR, it follows that if we prolong the horizontal lines II, BB, GG, we shall, after the second refraction, find RR' equal to SR, VV' equal to SV, BB' equal to SB, and so on; whence we conclude that the second image must be contained between the same horizontal lines with the first, without being at all dilated in length. The lower part of the first image, which suffers the least refraction, and is of a red colour, must also form the red part of the second image, and suffer also the least refraction. The same ratio must hold true for the opposite extremity, which is violet. Moreover, on account of the equality of SR and RR', the new image must be terminated laterally by two straight lines, inclined to the vertical VS, at an angle of 45°; and finally its axis $V'R'$, being prolonged, must pass through the centre S of the direct image. Now, by performing this experiment, we find that all these consequences are exactly realized, as Newton had assured himself. This agreement, therefore, confirms in the fullest manner, the unequal refrangibility of light, and it shows that this property of the rays is not accidental, but that it is inherent in their nature; since each of these preserves it invariably, after the first refraction, and even after the second and third, as Newton in like manner ascertained.

151. We have already remarked, that the size of the aperture through which the rays are admitted, must produce, even in the direct image of the sun, a penumbra or gradation of brightness, by which the borders of this image pass insensibly from the most vivid light to the most perfect obscurity. Now the feeble light which forms this penumbra, being absolutely of the same nature as the rest of the image, must be modified in the same manner by refraction, and consequently will be found on the rectilinear sides of the oblong image, whose distinctness

* We suppose the second prism, as well as the first, so placed as to make the angle of incidence equal to the angle of emergence. For it is only in this position, that an image formed of rays equally refrangible, is neither dilated nor contracted by refraction.

it thus affects. This is confirmed by experiment. In order to avoid this defect, Newton placed before the aperture in the window, a glass lens, which, by its refraction, concentrated nearly in one focus, all the rays sent from the several points of the sun's disc; and hence resulted an image of this luminary, white, circular, and perfectly free from penumbra. This being done, he received this collection of rays upon a prism placed behind the lens, and the oblong image formed by refraction, was found, in like manner free from every thing like penumbra on its rectilinear sides; so that these sides appeared as distinctly terminated as the direct image given by the lens.

152. In the experiment relative to lateral refraction above described, Newton found the means of operating separately upon each of the several rays of equal refrangibility. The following is another method employed by him of effecting the same purpose.

Let *SF* represent a beam of solar light introduced into the dark room by the aperture *F*. Near this aperture we place a prism *ABC*, which, refracting unequally the differently refrangible rays contained in this beam, will form on the screen *TT* an oblong, coloured image, as we have seen in the preceeding experiments. In order to examine separately the differently refrangible rays, of which this image is composed, we make in the screen *TT*, a very small circular aperture *O*, which answers to one of the points of its length. Then there will pass through this aperture a small cone of rays sensibly homogeneous, which, falling upon another screen *TT'*, parallel to the first, will form there a small circular image of the aperture *O*. This image will be of one single colour, as red, for instance, if the aperture *O* answers to the rays of the first image, which produce the sensation of red; and green, if it answers to the green rays, and so on. Now, when this homogeneous light is separated from the rest of the spectrum, we can examine it at pleasure. For this purpose we make at *O'* in the second screen, a small aperture, in such a manner as to admit only a small portion of the homogeneous light, and cause this portion after passing the aperture to pass through a second prism *a b c*. Then if the unequal refrangibility of the rays, contained in a natural beam, be the only cause of the elongation and colour of the image, nothing of this kind will here take place. The homogeneous portion, refracted by the second prism, will not have its colour changed; Fig. 106.

and if the inclination of this prism be such as to make the incidences and emergences equal, the image formed by this portion on the wall, will be round like the aperture itself. This is completely verified by the experiments of Newton and all succeeding philosophers.

153. If now we turn the first prism slowly about its axis, the rays which produce the sensation of the different colours, will pass successively through the aperture O, and will also arrive successively at the second prism $a\,b\,c$, exactly in the same direction OO', and consequently with the same incidence. We shall thus see formed successively on the wall, the image of red, yellow, green, &c., according as the rays which then pass through the aperture O, are found to be those which produce the sensation of red, yellow, green, &c. If then it is true that these rays are unequally refrangible, and that this property is inherent in them, they will experience in the second prism, unequal refractions, that is greatest for the violet, least for the red, and intermediate degrees for the rays which produce the intermediate colours; which may be easily perceived by the unequal height of the images, and by their rising or descending as we turn the first prism. All these consequences are perfectly confirmed by facts.

154. This unequal refrangibility of the luminous rays must necessarily produce its effect, when light is refracted by spherical lenses. For a lens is nothing but a circular assemblage of an infinite number of prisms of unequal refracting angles. Hence there must result for different rays, different focal distances, the rays most refrangible forming their focus nearest the lens, and those least refrangible, having their focus most distant. This is but too well confirmed by experiment; for the dispersion of the foci, thus occasioned, has been for a long time, a great obstacle to the perfection of telescopes.

155. Hitherto we have examined only those properties of rays which depend upon their unequal refrangibility. We come now to consider another property, discovered also by Newton; namely, the facility of the rays for interior reflection by refraction, is also unequal, and the more so, according as they are more refrangible. He conducted the experiment in the following manner.
Fig. 107. Having introduced through the aperture F, into a dark room, a beam of light FM, he caused it to fall upon the side AC, of a

prism *ABC*, whose two angles *B*, *C*, measured each 45°, *A* being a right angle. The refracted ray emerging at *M*, proceeded to form below the prism, a coloured image *VR*, as in the preceding experiments. By turning this prism slowly about its axis, in the direction *ABC*, the refracted rays are made to fall more and more obliquely upon the base *BC*, and the emergent rays also take a direction more and more oblique to this base. Hence, if we continue to increase the obliquity, the refracted rays will at length be no longer capable of emerging, but will be wholly carried back into the prism by refraction, as we have already explained. In common glass, this phenomenon takes place when the rays, in their interior incidence make with the base *BC*, an angle of about 48°. Then if the refracting angle *C* is 45°, as Newton made it, the incident rays will be nearly perpendicular to the first surface *AC* of the prism, and consequently the dispersion, caused by their first refraction, is nearly nothing; so that their interior incidences upon *BC* are sensibly equal. But notwithstanding this equality, their interior reflection is progressive; for if, during the motion of the prism, we observe attentively the coloured image *VR*, we shall see that the violet first disappears from this image, while the other colours remain. After the violet, the blue disappears, then the green, then the yellow, and finally the red, which disappears last. It is then only, that the interior reflection is total. These rays, successively reflected, emerge through the side *BA* of the prism, to which they are also nearly perpendicular. If they were the only ones which followed this direction, we should thus have a very simple means of separating them wholly from the rest. But this is not the case; for the refracted beam experiences always at *M* a partial reflection, which, acting without distinction upon all the incident light, sends a certain proportion of it directly towards the face *AB* of the prism; so that the coloured parts of the portion *VR*, which come next to be reflected, only add to those already reflected. Still we can here recognise their influence. For this purpose we first place the prism *ABC*, in such a position, that the refracted beam *VR* shall emerge entirely; then in the direction *MN* of the returning beam, produced by partial reflection, we place a second prism *A'B'C'*, which, refracting it, forms upon the screen *TT*, another oblong and coloured image *R'V'*. We then observe attentively the intensity of this image; and if

we turn the first prism *ABC*, slowly about its axis, in such a manner as to increase gradually the obliquity of the rays refracted at its base, we shall see that at the instant when the violet rays can no longer emerge through *BC*, the violet part of the image *R'V'*, will receive a very sensible increase of intensity, compared with the other tints which compose it. Next the blue will increase, then the green, the yellow, and finally the red, when, by the continuing the motion of the prism, the reflection becomes total at *M*. Newton varied the experiment in several ways, and the result of the whole was what I have above stated.

156. In all this variety of experiments upon light transmitted through natural bodies, upon light reflected from specular surfaces, and lastly, upon refracted light, we always find rays which, at equal incidences, (the medium being the same) suffer unequal refractions, although no dispersion takes place in each simple ray. We find, moreover, that this effect is not produced accidentally by imperfections in the refracting substance; but that it is governed by regular laws, depending upon the position of the refracting prism, the refracting angle, and the nature of the substance. From all this, it follows incontestibly, *that the light of the sun, like other kinds of light, which can be subjected to the same experiments, is a mixture of heterogeneous rays, of which some are constantly more refrangible than others, and which, considered separately, are capable of producing upon our organs the sensation of different colours.* Moreover, since the violet rays return into the prism, under interior incidences at which the others emerge, we may add, *that these rays differ also in reflectibility, and that those which are most refrangible are also most susceptible of being reflected inward by refraction.* According to the theory of attractive forces, this is a consequence of unequal refrangibility.

Another consequence of this inequality is, that the solar spectrum, as given by a prism, is really nothing but the succession of an infinite number of small circles, partly superposed upon each other, and having each a simple colour, from the violet end to the extreme red. This is represented in figure 108, in which it is only necessary to suppose an infinite number of successive circles, from the violet to the extreme red, instead of the small number here represented.

If the image thus dispersed be that of the sun, whose apparent diameter is about half a degree, each of these several circles, seen from the centre of the circular aperture, supposed to be infinitely small, subtends also an angle of about half a degree. For each kind of simple rays, coming from the opposite limbs of the sun, forms, in passing through the aperture, a cone whose angle at the centre is equal to the apparent diameter of the sun's disc; and when refracted in the prism, with the angles of incidence equal to the angles of emergence, the refracted cone has sensibly the same angular opening as the direct cone. The size of these consecutive images, causing them necessarily to lap upon each other, it follows that, strictly speaking, light is nowhere absolutely homogeneous, except on the rectilinear sides of the image where the circles are detached from each other.

The means of simplifying the effect, is then to diminish the diameters of these circles, in order to separate them more from each other, the distance between their centres still remaining the same. This we do by greatly contracting the aperture by which the rays enter the dark room, or by giving it the form of very narrow slit, having its small diameter in the direction of the refraction. Among the many precautions employed by Newton to obtain this separation, one of the most important, is to concentrate the beam by a lens, before it falls upon the prism. We should take care also to place without the aperture, in the direction of the solar beam, a narrow and black tube, in order to exclude the foreign light, which, coming from the lateral parts of the heavens at a great distance, would form in the dark room large cones, and consequently, present after refraction, large circles, which mixing with each other and with the principal image *VR*, would affect the distinctness of its colours.

157. It is upon light thus rendered pure, that the decisive experiment on the immutability of colour, illustrated by figure 106, should be made. This remarkable property is then preserved, not only after one refraction, but after all the successive refractions which the homogeneous beam is made to undergo. It is not destroyed even by reflection; for if insects or other small objects, be placed in this light, and examined through a prism, even of a great angle, we see them just as distinctly as if we viewed them directly, and with the single colour that illuminates them; whereas we could not distinguish them at all, if we looked at

them in this manner, when they were illuminated by the compound light of the sun, because the image of their different parts would be elongated and deranged in consequence of the unequal refrangibility of the different rays. We might make the same experiment with a printed book. If it is illuminated by homogeneous light, the characters can be read with perfect distinctness, however fine the impression is; while they cannot be distinguished at all after reflection, if they are illuminated with compound light.

If we consider the course of luminous rays, thus purified, through the same refracting medium under different incidences, we find, as before remarked, that the sine of refraction is always in a constant ratio to the sine of incidence. We shall hereafter have occasion to note the most delicate proofs of this fact. For the present, we only observe that, according to the theory of the emission of light, it becomes a necessary consequence of the affinity existing between luminous particles and refracting bodies.

158. The reason why refraction causes certain luminous particles to deviate more than others, cannot be assigned with any degree of certainty. We might be induced to believe that this inequality is owing to a difference as to their mass or velocity; but in this case bodies which refract the one class of rays equally, ought also to refract all the other classes equally, which is not the case, as will soon be shown by direct experiment. For instance, there are some bodies, which refract the green rays as little and even less than other bodies, while they refract the violet rays more. We are obliged, therefore, to believe that the chemical nature of the particles of light, and perhaps their form, contributes to this phenomenon, and causes the affinities of these particles, while traversing different bodies, not to preserve among themselves the same constant relations.

159. We have seen that certain crystallized bodies have the singular property of dividing into two portions the luminous rays which traverse them, and have called this phenomenon double refraction. In this case, each of the two refracted portions becomes also dispersed. In crystals of one axis, where the velocity of the ordinary ray is constant, the general law of its dispersion is the same as in bodies which produce only simple refraction. The law of dispersion of the extraordinary ray is more complicated, because it is affected at the same time by the

ordinary refracting forces, and the repulsive or attractive forces which emanate from the axis of the crystal. In crystals of two axes, this complexity extends to the two velocities. We shall examine the combined effects thence resulting, when we have completely established by experiment, the phenomena exhibited in ordinary dispersion. For the present, it is sufficient to remark, that the refrangibility and colour, peculiar to each ray, are not altered or changed by double refraction.

160. Since each species of rays of a particular refrangibility, retains thus invariably its own peculiar colour, we may, for the sake of brevity, designate each ray by the species of colour of which it produces the sensation. Thus we denominate red rays, those which are least refrangible and which produce the sensation of red; yellow rays, those which produce the sensation of yellow; and violet rays, those which produce the sensation of violet; not because we suppose that these rays are really red, yellow, and violet, or that they have colour in themselves, any more than sonorous bodies contain sound; but only to express the sensation which they are capable of producing, and which they do constantly produce in eyes properly organized.

Nevertheless it is impossible to assign, in this manner, particular denominations to all the rays; for since each ray endued with a different refrangibility, produces on our organs the sensation of a peculiar colour, the number of shades following each other in the spectrum, must be infinite like that of the rays which produce them. But since the most accurate eye cannot have a distinct sensation of so many shades, differing so little from each other, it is sufficient to establish among them, a certain number of divisions, which, comprehending all the shades in their several intervals, will enable us to fix their place and character with a precision corresponding to that of our senses. This was done by Newton; he drew seven principal lines of separation in the spectrum between the extremes of the greatest and least refrangible rays. The divisions thus formed, being designated by the colours belonging to them respectively, are the following; *violet, indigo, blue, green, yellow, orange, red.*

The rays comprehended in each of these divisions, may be regarded as homogeneous, as to refrangibility and colour; and each portion retains its peculiar refrangibility and colour, whatever refraction it is made to undergo; and, what is no less

are expressed in degrees,* we shall have;

$$
\begin{aligned}
RO &= 60^\circ\ 45'\ 34'' \\
OY &= 34\ \ 10\ \ 38 \\
YG &= 54\ \ 41\ \ \ 1 \\
GB &= 60\ \ 45\ \ 34 \\
BI &= 54\ \ 41\ \ \ 1 \\
IV &= 34\ \ 10\ \ 38 \\
VR &= 60\ \ 45\ \ 34.
\end{aligned}
$$

Let us now consider these different arcs in the order in which they follow each other, as representing the seven principal colours of simple light, which compose the spectrum; so that the entire circumference will represent the whole series of shades through which this light passes, from the first rays of red to the last of violet. Then, having determined the centres of gravity r, o, y, g, b, i, v, of all these successive arcs, suppose in each of them a weight proportional to the corresponding arc; and conceive these weights to be so many forces tending to draw to themselves the centre C, and the eye supposed to be placed there. According to this supposition, it is evident that the eye will remain at rest, being placed at the centre of gravity of all the weights; and this rest will correspond to the perfect white produced by the simultaneous sensation of all the shades of simple light, when they are mixed together according to the proportions in which they exist naturally in the spectrum. But suppose these proportions changed, as they always are in a coloured mixture which differs from white; then it will be necessary to place in each partial centre of gravity, not the total weight of the corresponding arc, but the half or third, or generally the n part of this weight, according as the given mixture contains the half, the third, or generally the nth part of all the light which composes this colour in the spectrum. This being done, if we find the common centre of gravity of all the partial

* This division requires only the simple rule of fellowship. As the sum of all the fractions is to 360°, so is one of the fractions to the arc which corresponds to it. The sum of the fractions is $\frac{3}{9} + \frac{2}{10} + \frac{2}{16}$ or $\frac{79}{120}$. If, for example, we would find the arc corresponding to $\frac{1}{10}$, its value will be $\frac{\frac{1}{10} \cdot 360^\circ}{\frac{79}{120}}$, or 54° 41′ 1″.

weights, it will, in general, no longer coincide with the centre of the circle. While it does coincide, the colour of the mixture will still be white; but when, on the contrary, it approaches considerably one of the partial centres, the mixture will present, as the predominant colour, that which belongs to this centre. Finally, wherever it falls, as in G', for example, we have only to draw from the centre to this point the line CG'; then the direction of this line will indicate the predominant colour of the mixture, and its length, or the distance from the point G' to the centre C, will indicate its vividness. If, for example, CG' is situated exactly midway between CY and CG, the compound colour will be the most perfect yellow; but if CG' approaches more to CG or CY, this yellow will incline more to the orange or the green. On the first supposition, if the point G' falls very near the circumference, the colour will be strong and vivid in the highest degree; if it falls half way between the circumference and the centre, the intensity of the colour will be diminished one half, and it will be that of a mixture of the liveliest yellow with an equal quantity of white. In general, if we represent the distance CG' by Δ, the radius of the circle being 1, $1 - \Delta$ will express very nearly the proportion of white which it will be necessary to employ, in order to imitate to the eye the colour in question; this white being mixed with the proportion Δ, of the single colour towards which CG' is directed. In this manner, then, the nature and intensity of the colour are ascertained. It is proper, however, to remark, that if the point G' falls upon the line CR, or very near this line, the red and violet being then the principal elements of the compound colour, it will no longer correspond to any of the prismatic colours, but will be a purple, inclining to the red or violet, according as the point G' varies from the line CR, towards the one or the other of these colours; and in these two cases, the purple or mixed violet, will be more dazzling and brilliant than the simple violet. In general, we observe the greatest analogy between the effects of the violet and the red; so much so that we can form with blue and red, mixtures which produce on the eye the sensation of a fine violet; and it is probably this kind of return of tints upon themselves, perfectly analogous to the consonance of octaves, which led Newton to compare, as he has frequently done, the impressions produced by the different colours to those produced by the musical intervals.

170. According to the extent assigned by this rule to the arcs which represent the seven principal shades of the spectrum, there are no two whose centres of gravity are diametrically opposite to each other. Accordingly, in whatever proportion two colours are mixed, their common centre of gravity can never fall upon the centre of the circle; and, therefore, we infer that they can never form a perfect white, but only a pale tint, nearly approaching to white. This inference is confirmed by experiment. "For," says Newton, "I have never yet been able to obtain a pure white, from mixing only two primitive colours. I do not know that it can be done even with three colours. But these are points of curiosity, which have little or nothing to do with the understanding of natural phenomena; for, in all natural whites, there is ordinarily a mixture of all the primitive colours, and consequently these whites are compositions of all these colours, which circumstance confirms the propriety of the mode of composition just described."

171. We may make use of this rule of Newton, to produce the sensation of white by a rapid succession of the different colours. We have only to divide a pasteboard circle according to the proportions above indicated, paint the several divisions with the purest colours that can be obtained, and then turn it rapidly about its centre. But as the colours thus employed are never simple, we must not expect to obtain by this process a perfect white.

Influence of the Unequal Refrangibility of Rays of Light upon Vision through Refracting Surfaces.

172. From the time that the phenomenon of dispersion presented itself in our experiments on refraction, we have foreseen the general effects which it must have upon vision through triangular prisms. The knowledge we have now acquired of the individual properties of simple rays, and the constancy of their colorific powers, affords a full confirmation of what was thus indicated, and enables us to state it with more precision.

When an infinitely small luminous point is seen by refraction through a triangular prism, each kind of simple rays which proceeds from this point gives an image tinged with its peculiar

colour. The unequal refrangibility of the rays of different kinds causes the images to appear separate and placed by the side of each other in the order which their colours take in the spectrum, those formed of the most refrangible rays being most deflected. But, if instead of a single radiating point, infinitely small, we look at several points placed by the side of each other, each of them will be found to produce a similar spectrum; but these spectrums admit, to a certain extent, of being placed one upon the other, in such a manner as to produce white in the place where they unite. This usually happens when we thus look at objects of a sensible magnitude. If the surface of the objects is white and equally luminous, the interior portions of the refracted image appear white, and the colouring of the image is perceptible only at the extremities, in the general direction of the refraction.

Newton states a fact, that is easily observed, which, at first sight, seems to be inconsistent with this theory, but is in reality a consequence of it. Having placed horizontally, before an open window, the base of a prism *ABC*, bring the eye to *O* so as to receive the light of the sky reflected inward from its base. Then, when the reflected rays form with this base an angle of about 50°, there will be seen an arc of a blue colour of a certain width *SS'*, which will extend the whole length of the prism, upon turning its concavity towards the eye; and the portion of the base *SB*, situated beyond this arc, will appear much more brilliant than the portion *S'A*, which is on this side of it. Fig. 113.

In order to explain this singular phenomenon, let us conceive that the angle of reflection *NSI*, which corresponds to the convex part of the arc, is the smallest angle at which total reflection commences for the red rays, which are the least refrangible; they cannot then be wholly reflected under an incidence approaching nearer to the normal *SN*. Let us suppose also that *N'S'I'* is the smallest angle of total reflection for the last violet rays; then from *B* to *S* there may be rays of all kinds which will undergo a total reflection and arrive afterward at the eye; but on this side *S*, the interior incidence becoming too near to the perpendicular, none of the extreme red rays can be perceived; thus, as the point of interior incidence approaches *A*, the possibility of total reflection towards the eye will cease successively for the different rays, one after another, to *S'*, when this possi-

bility will cease, even for the last rays of the violet colour; so that from S' to A total reflection will not take place for any of the rays. If we now examine these results, we shall see that from B to S the reflection will be vivid and brilliant, because it will be total, and will comprehend the rays of all the colours. From S to S' the total reflection will go on diminishing, and will be limited to the more refrangible rays, which must produce a zone where the colours of these rays are predominant; finally, from S' through every other point nearer to the eye, the reflection will not be total for any of the rays. The eye will then receive from this space only the feeble portion of light which arises from radiant reflection; it will of course appear obscure in comparison with the intense brightness of BS. The limits of all these phenomena may be fixed by the aid of the Calculus.*

We see, also, that the remainder of the rays which fall on the space SS', passing out below the prism, must produce, after emergence, an arc in which the red rays are predominant,
155. very nearly, as in the experiment of Newton, already mentioned.

173. This theory furnishes also an explanation of the phenomena of the rainbow, and enables us to calculate these phenomena. It is well known, that this meteor presents one, and sometimes two arcs, tinged with all the colours of the spectrum. It is never produced, except when there is a fall of rain accompanied with sunshine; but the union of these two circumstances is not sufficient of themselves to cause this appearance; we require also a particular position of the cloud, of the observer, and of the sun, which must always be directly opposite to the centre of the bow. These circumstances long since led to the belief that the rainbow was produced by the refraction of light in the drops of rain; indeed a similar appearance is exhibited in that kind of artificial rain which is caused by jets and cacades, especially when agitated by the wind. In order to conceive in what manner the light is capable of being thus dispersed by refraction, let us sup-
Fig. 114. pose a single ray of light SI, infinitely small, coming from the sun, to fall upon a single spherical globule of water, and let us follow the course of this ray. It will first undergo a refraction at I, which will direct it to I', where part of the ray will be a

* See Biot's *Traité de Physique*.

second time refracted, and will emerge into the air in the direction $I'R'$, but the rest of the ray will be reflected into the interior of the globule toward I'', where a similar effect will be produced, that is, a part of the ray will emerge into the air, while the other portion will be again reflected inward to I''', at which point the same division will take place, and so on indefinitely. This explanation may be verified by introducing a small solar ray into a dark room by means of a heliostat, and directing it through a glass cylinder filled with water. The course of the ray in the interior of the water will become visible by the partial reflection of light, occasioned by the particles of the water, and the successive emergences will also become sensible by placing the eye, or unpolished glass in the direction of the emergent rays. In this manner it will be seen that the rays are dispersed, presenting the series of prismatic colours; their intensity is diminished according as they have undergone a greater number of divisions, and at last the ray will become so feeble as to be insensible.

174. The phenomenon of the rainbow is caused by coloured spectrums, which emerge from the different drops of water after two refractions, separated by one or two intermediate reflections. Let us now inquire how the superposition of these partial spectrums produces the colours of the rainbow and determines its magnitude. In order to do this in a simple manner, let us take one simple incident ray, as red, for example. If this ray, after being refracted in the globule, is reflected once or several times at the second surface, and then emerges into the air, it will make, after emerging, a certain angle, with its primitive direction. This angle will be constant for all the rays of the same nature, which enter the globule under the same incidence; but it will vary with the incidence. In order to comprehend these variations, let us first consider the case in which the ray suffers only one interior reflection before emerging. Then, if we calculate numerically Fig. 115.
the quantity of deviation for several incident parallel rays on the surface of the globule, the deviation at first is nothing, when the incidence, being perpendicular, the ray passes through the centre of the globule; then, the deviation gradually increases until the angle of incidence has reached a certain limit, which is about $54^\circ\frac{1}{2}$ for the red rays. Accordingly, a small parallel beam of these rays entering the globule at I, under this incidence, and being reflected once at the bottom, emerges also a par-

allel beam at I'', although the general direction of the ray will have deviated 42°. But for greater incidences the deviation diminishes as it before increased, and this diminution continues to the last rays, which are tangents to the globule. Now, if we intercept the rays at so great a distance from the globule that it may be considered as a point, it is evident, that all those which correspond to the unequal deviations, will depart more and more from each other, according as their distance from the globule is increased, so that they will at last be too feeble to produce the sensation of the globule in an eye placed in their direction; while the eye may be affected, and indeed is affected at any distance whatever, by the emerging rays which answer to the maximum of deviation, because, being parallel to each other they may pass to any distance without being separated. The effect will even be so much the more vivid according as, the density being uniform at their incidence, they are compressed and condensed, as the calculus shows that they are when they emerge in the manner above stated. Let us then suppose that a number of such globules are arranged circularly in such a manner that the refracted rays which come from them, and which are supposed to be of a uniform colour, shall arrive at the eye of the spectator. They will give the sensation of a luminous line; and several such rows, placed side by side, will produce, on account of the sensible opening of the pupil, a coloured space which will be equal to it in width.

The same remarks apply equally well where the reflections and refractions are more numerous. There is, in every case, a certain limit of incidence, at which the rays coming from the same beam and emerging near to each other, will leave the globule sensibly parallel, so that they may be transmitted to a great distance without becoming less vivid.

175. In order then to develope the consequences of what we have now established, we will suppose a person, placed at O, to Fig. 116. look at a vast cloud, consisting of a multitude of spherical globules of water. Through the eye of this observer we draw the line SOC to the centre of the sun, to designate the direction of the incident rays, which we will first suppose to be parallel, or, which amounts to the same thing, to proceed from a point at an infinite distance. This being done, there will take place at the first surface of the globules, a partial reflection of all the colours

composing the radiant light, which will impart a whitish tinge more or less deep, to the whole extent of the cloud. Besides this, if the cloud is sufficiently extended, two concentric arcs will be seen, tinged with all the colours of the spectrum. For if, through the eye at O, we draw the line OV, forming with OC an angle of 40° 17′, and cause it to revolve round OC, describing a conic surface, all the globules in the prolongation of this surface will be exactly in the situation necessary, in order that the most refrangible violet rays, which have undergone two refractions and one intermediate reflection, may emerge parallel and arrive at the eye in O, and this will not take place in the same manner on any other part of the cloud; so that by means of this species of rays, the spectator will see on the cloud a violet coloured arc, of which OC will be the axis, and C the centre. He will, moreover, behold an infinity of other concentric arcs exterior to the above, each of which will be formed of a single species of rays; and, according as these rays are less refrangible, their arcs will have a greater diameter, so that the largest, composed of extreme red, will subtend an angle ROC of 42° 2′. Thus the whole width of the coloured bow will be 42° 2′ — 40° 17′, or 1° 45′. The red will be on the outside and the violet within.

The contrary order will be observed in the case of two reflections. If we draw through the eye, the lines OR', OV', making with OC angles of 50° 59′, and 54° 9′, and with these respective inclinations cause them both to turn round OC, as an axis, the first will meet with all the globules, which, after having twice refracted and twice reflected the extreme red rays, can transmit them to the eye parallel to each other. The second will give the corresponding limit for the extreme violet rays. Between these two arcs there will be others tinged with all the intermediate prismatic colours. The whole taken together will form another bow, which will have for its width 54° 9′ — 50° 59′, or 3° 10′. This bow will have its colours disposed in the inverse order of those of the first, that is, the red will be on the inside, and the violet without. The distance of the two red arcs will be 50° 59′ — 42° 2′, or 8° 57′.

176. Such, therefore, would be the dimensions and distance of the rainbows, if the sun were but a point. But the sun has a sensible apparent diameter, which may be supposed, at a mean,

to be about 30′. Hence, if we consider the arcs we have determined, as produced by rays coming from the centre of his disc, the rays which come from the borders and from the interior, will form so many similar arcs of the same size, but which will have each for its axis the line drawn from the observer's eye to the point on the disc of the sun from which they emanate.

Consequently, if we describe from the point C the circumference of a circle $C'C''C'''$, equal to the apparent diameter of the sun, as seen from the point O, there will not only be formed around the centre of this circle an interior violet arc at the distance of 40° 17′; but there will also be as many of these arcs as there are points in the circle $C'C''C'''$, which are capable of becoming centres in their turn; that is, there will be formed a circular violet arc of a width equal to the apparent diameter of of the sun, and which will have its interior radius equal to 40° 17′ — 15′, or 40° 2′, and its exterior 40° 17′ + 15′, or 40° 32′. In like manner, the red arc which we found to be 42° 2′ from OC, will become a red bow, whose exterior border will have for its radius 41° 47′, and its exterior border 42° 17′; thus the whole width of the rainbow, comprised between these two extremes, will be 42° 17′ — 40° 2′, or 2° 15′, greater by 30′ than if the sun had been only a point. In like manner, the width of the exterior iris, which we have fixed at 3° 10′, will become 3° 40′; its interior semidiameter which was 50° 59′, will become 50° 44′, and the exterior, which was 54° 9′, will become 54° 24′. Finally, the distance between the two bows which was found to be 8° 57′, will be reduced to 8° 27′. But, owing to the width and the superposition of the partial arcs which compose them, the colours will be much less distinct than in the former case. Still these dimensions, determined by calculation, are exactly conformable to those furnished by observation, at least when the colours are the most vivid and the most strongly marked. For Newton, having on a certain day measured a rainbow, by means of instruments which he then possessed, found that the exterior semidiameter of the interior iris was about 42°, and that the red, the yellow, and the green, of this bow, taken together, were about 63′ or 64′, besides three or four minutes that may be added on account of the exterior red rays which were enfeebled and obscured by the brightness of the neighbouring clouds. The width of the blue was about 40′, at the least, with-

but reckoning the violet, which was so obscured by the light from the clouds, that its width could not be measured. But upon the supposition that the width of the blue and violet, taken together, was equal to that of the red, yellow, and green, united, the whole width of the interior iris would be about 2° 15′, as above stated. The smallest distance between the two bows was 8° 30′. The width of the exterior iris was greater than that of the interior; but its hues were so feeble, especially on the blue side, that it could not be measured with accuracy. On another occasion, when the two bows were distinct, Newton found that the width of the interior iris was 2° 10′, and that, in the exterior, the width of the red, yellow, and green, was to that of the same colours in the interior, as 3 to 2.

177. In that part of Newton's optics where this admirable theory is unfolded, the first idea of the explanation of the rainbow is attributed to Antonio de Dominis, archbishop of Spalatro, who, he remarks, confirmed it by observations made upon a glass ball, filled with water, and placed in different situations with respect to the sun and the eye of the observer. Newton adds, that Descartes corrected the observations of the exterior rainbow. But he had probably never read the work of De Dominis, for if he had, he would have perceived that this prelate had formed a vague notion that the rainbow might be the effect of refraction in the drops of water, without attempting to confirm this idea by the experiments of which Newton speaks; and his mode of explaining the formation of the meteor is without reference to the true theory. The experiments really belong to Descartes, and to Descartes alone. This philosopher did every thing that could be done, at a time when the unequal refrangibility of light was unknown. He determined, by numerical calculation, the course of the rays which enter the drop of water, and emerge thence after one or more reflections. By this calculation, he discovered that of all the rays that enter the drop, those only which fall upon it at a certain inclination can come to the spectator without diverging from one another, and being consequently enfeebled. He thus ascertained the true circumstances that are necessary to the existence of the rainbow, and he proved by experiment, that they were conformable to actual observation. It only remained to assign the cause of the colours. Descartes, unacquainted with this cause, reduced it to

another more simple case, that of the decomposition of light by the prism; and he proved that the part of the drop of water in which the light is refracted, must disperse this light, in the same manner as a prism of water, with plain faces, whose angle of refraction is equal to that formed between two planes tangent to the points of the drop where the rays enter and emerge. He confirmed this theory by a very circumstanstial experiment, which is given in the *Traité de Physique*.

178. Besides the rainbow, other meteors are frequently observed in the atmosphere, as the large whitish circles or haloes, that are seen to surround the sun and moon, commonly at the distance of 45°, and parhelia and paraselenes, false suns and false moons, attended with other remarkable appearances. A particular description of these phenomena, with the hypothetical explanation of Huygens, may be found in the *Traité de Physique*.

Achromatic Combinations.

179. When two or more prisms are so constituted and disposed as to render colourless the images of objects seen through them, the combination is called *achromatic*. If these prisms were all of the same substance, the opposite refractions would balance each other, when their angles were equal and their directions contrary, so that they would together form a plate with parallel faces. Rays traversing such a system of prisms would continue on in their primitive direction. But if substances could be found capable of producing equal dispersions, with unequal refractions, a system of prisms might be formed which would give images of objects free from colour, the rays being deflected at the same time. Accordingly, by transferring this principle of compensation to the curved prisms which form the borders of spherical lenses, we might obtain compound object-glasses, which, with a limited focal distance, would produce colourless images of objects; and we might employ them in the construction of telescopes incomparably superior to those provided with single object-glasses. Are there then in nature substances which thus compensate each other? If not, can their place be supplied by an approximate compensation? And in this event, upon what prin-

ciples is this approximation to be established? How can it be produced with given substances? Is it more easily produced by some substances than by others? These are the questions which the subject presents for our examination.

180. Accordingly, let us first consider a perfectly homogeneous ray *SI*, which, after traversing a certain number of prisms, of whatever substance and angles, arrives at last at the eye of the observer situated in *O*. From this point let us draw a line *OS* to the object from which the ray proceeds, and which, for greater simplicity, is supposed to be removed to an infinite distance, so that the line *OS* may be considered parallel to *SI*. It is evident, that if this direction is given, together with the positions of the prisms, their substance, their angles, and the place of the observer, the angle of deviation *ROS*, comprehended at the eye between the primitive direction *OS* and the emergent ray *OR*, may be determined by calculation. Let us suppose now, that the ray, instead of being simple, is compound, like the white ray. Then, after traversing the system of prisms, it would be dispersed into an infinity of different rays, having different directions $v\,v'$, $o\,o'$, $r\,r'$, from the violet to the red; so that the observer, placed in the direction of one of these rays, would perceive only that single ray. But if we conceive an infinite number of white pencils of rays, all proceeding from the same point, to arrive together at the first surface of the prism, the observer, stationed at *O*, would receive all these different species of simple rays, through different points of emergence; which will be found by drawing from *O* the lines OR', OO', OV', respectively parallel to the lines $r\,r'$, $o\,o'$, vv', and terminating in the same manner upon the surface of the last prism; for these rays, being turned back through the successive prisms, each with the proper degree of refraction, will emerge in a direction parallel to the incident rays, and consequently, will make a part of the incident light. Then, in order to take from the observer the sensation of these separate rays, we have only to dispose the system of prisms in such a manner, that the rays shall all emerge parallel to each other; for then the angles $O'OV'$, $O'OR'$, &c., with which they cross each other at the eye, becoming nothing, the points of incidence V', O', R', will coincide, and the pencil will consequently enter the eye free from colour.

Fig. 117. Fig. 118

181. From this analysis it will be perceived, that the rigorous determination of the achromatic effect involves for each ray a particular condition which must be satisfied by the system of prisms. Hence it follows, that as there is an infinite number of rays of unequal refrangibility, so there are, strictly speaking, an infinite number of conditions which cannot all be fulfilled, because the number of prisms must necessarily be limited. The problem does not therefore admit of a rigorous solution. But a solution will be obtained with sufficient accuracy, if instead of attempting to render all the different rays parallel, we only render a few of them parallel to each other, which may easily be accomplished by a small number of prisms. Indeed, if we bring together in this manner the two extreme rays, the violet and the red, or two extreme rays and one mean, as the red, the violet, and the green, the removal of colour for the intermediate rays will manifestly be so far accomplished, that no sensible effect will be produced upon the eye.

This is in fact the course to be pursued; and it will be easily perceived that two rays may be united by two prisms, and in general, as many rays as there are prisms employed. If but two prisms are employed, they should be joined base to vertex, as in figure 119, so that their refractions may be in opposite directions; indeed, if the two prisms are of the same substance, we have seen, that in order mutually to compensate each other, they should be placed in this position with equal refracting angles.

182. But when the substances are not of the same nature, how are we to determine the ratios of the angles of refraction with which a compensation must take place? We can manifestly approximate to this determination, by considering the magnitude of the spectrum furnished by prisms of these two substances formed with equal angles. We can then effect the proposed object still more nearly by gradually diminishing the power of that which causes the greater degree of dispersion, and which, when the two prisms are combined, colours the image in the direction of its own dispersive power. When the compensation is thus in part effected, we can easily discover the best of all possible compensations by the following process, which was applied, in a great number of instances, with entire success, by M. Cauchoix and myself.

At the extremity of a good achromatic telescope, and parallel
Fig. 120. to its axis, are placed two metallic rods T, T, diametrically oppo-

site. These are pierced transversely with two holes *A*, *A*, through which pass the axes of two copper frames C, C, that turn freely round the straight lines *A*, *A*, and perpendicular to the axis of the telescope, the motion being measured by graduated circles. To these frames are attached the two prisms that are to be compensated, and the apparatus will then be disposed, as represented in the figure. In order that the experiment may succeed completely, the angles of the prism must be such that when they are turned in opposite directions, they will nearly form an achromatic system; this can be accomplished by repeated trials, as mentioned above. It has been already shown, that the dispersion does not depend simply on the angular opening and chemical nature of the substance; it varies also with the incidence of the rays; and this incidence enters as an element into the analytical conditions to be fulfilled, in order that the rays of different kinds may be parallel after their emergence. We can avail ourselves of this variation, in seeking, by experiment the precise situation in which the compensation is most perfect. Accordingly, let us suppose, that the telescope and the two prisms are so disposed, that a white object at a great distance may be seen through the system. If the image of this object is found to be equally white throughout, the prisms are properly placed; and this is the position which renders the achromatic effect complete; but it is not to be expected that a colourless image will be obtained on the first trial. A very sensible colour almost always presents itself after the refracting angles of the prisms are opposed to each other. Then we turn slowly one of the prisms in the direction necessary to diminish the colour. If a motion of one of the prisms is not sufficient, both must be moved successively, and thus we shall at length find the position in which the compensation is the most perfect. The prisms are to be fixed in this position, and it will only be necessary to measure the angles they make with each other, and with the rays that traverse them. Hence the design of the graduated circles that serve to measure the angles passed over by the frames. There are some precautions to be used for determining the precise point of each division which renders the frames perpendicular, respectively, to the axis of the telescope, and also for arranging the prisms in such a manner, that their refracting angles shall be exactly compre-

hended in the same plane. In the experiments made by M. Cauchoix and myself, the object selected as a mark, was a strip of white paper pasted to a sheet of black pasteboard; its width was about four inches, and its length about three and a half feet. We fixed the pasteboard in a vertical position, at a distance of about twenty rods, so as to present the longer sides of the piece of paper in a horizontal direction; and we placed the prisms in such a manner, that the planes of their refracting angles, in which the dispersion took place, should be vertical, which was effected by directing them towards the edge of some edifice or tower at a distance, turning them on their frames, till they ceased to exhibit any lateral deviation. When a proper position was found, they were made fast, and the foot of the telescope was also secured by a screw, preventing any horizontal motion. These preparations are of great importance in facilitating the achromatic process. For, when the image of the object is held in the telescope, it cannot be displaced, except in a vertical direction. Hence, while, by gently turning the prism with one hand for the purpose of destroying the colour, the image is displaced, it may easily be brought back by raising or depressing the telescope with the other. If the refracting angles are so disproportionate that the effect of colour cannot be corrected by a mere change of incidence, it will be easily perceived; for, if we suppose, for example, that the second prism is too powerful, and that the vertex of its refracting angle is directed towards the earth, then this prism, if it acts alone upon the light, will give a coloured image of the object, in which the red rays, being least refrangible, will appear highest, and the violet rays, which are most refrangible, lowest. Consequently, if the image of the object, seen through the two prisms, presents constantly this arrangement of the colours, in every variety of situation, it is a certain proof that the dispersion of the first prism, which acts in a direction contrary to the other, can never be sufficient to compensate it, much less to exceed it. It follows therefore, that the diminution of colour that can be produced by this system, is far from the greatest possible. In such cases it will be necessary to diminish somewhat the refracting angle of the second prism, which has thus been found to be too great, and then to replace it. If it has been sufficiently diminished, upon moving the two prisms alternately, we shall

obtain images of the object in which the red fringes are turned upwards, and other images in which they are turned downwards. In the first case, the posterior prism prevails, in the last, the anterior. Between these two opposite states there will be found one or more situations, in which the colours of the borders of the image will be the least possible. Such combinations afford the most favourable compensations. We preserve that combination which seems best, that is, which gives the shortest and especially the dullest fringes. The prism and the telescope are then made fast in their places by means of screws. Special care should be taken to avoid the red and yellow fringes; for these two colours being brighter than the others, have a much more sensible effect when these results are applied to the construction of telescopes, for which they are principally designed. For a contrary reason, we should give the preference to those positions which produce fringes of obscure and deep colours, those, for example, of a brick red, or of a greenish blue; for these colours being less vivid, their effect will be less conspicuous in telescopes, and it will be altogether insensible in observations made at night.

183. Having found the most favourable disposition, we observe the positions of the two prisms by means of the graduated circles; from these we deduce the angles of incidence and emergence of the mean rays, and these elements, introduced into formulas, determine the ratios of the refracting angles under which the compensation of the two prisms will be effected, when placed upon each other, and exposed perpendicularly to the luminous rays. In our first experiments upon dispersion, we remarked that each prism had a certain position in which the refracted image becomes stationary; and that before and after this limit, the refraction and dispersion increase simultaneously. Hence in experiments upon this subject we ought to find for each prism several positions, in which the same effect is produced upon the image transmitted. This is indeed the case. But upon calculating the course of the rays, for all the various positions, on the supposition of a constant ratio of refraction for the different angles of incidence, we arrive finally at the same ratio of compensation, and that too with surprising accuracy, as may be seen in the *Traité de Physique*. This agreement is a very delicate and certain proof of the constancy of the ratio of refraction for each simply ray.

184. Whatever care we use in these experiments, if we employ a telescope which magnifies fifty or sixty times, and prisms whose refracting angle is at least fifteen degrees, conditions which are necessary in order to render the coloured fringes sensible, it will soon be perceived that an entire removal of colour is altogether impossible, except in the single case in which the two prisms are composed of one and the same substance. For, when the prisms have those refracting angles that are very near the proportion, which produces the best achromatic effect, each colour may be made to pass successively from one side of the image to the other without an intermediate position being found in which the coloured fringes disappear entirely. From this impossibility we infer with the utmost certainty, that the dispersion of the rays does not by any means take place according to the same laws in substances, which differ in their chemical composition; we infer also, that when two homogeneous rays, of whatever kind, have been refracted in such a manner as to be parallel at their emergence from the two prisms, the homogeneous rays of different species will be inclined to each other, and will form fringes on the borders of the object. Hence it is evident, that in order to obtain more perfect compensations, more than two prisms must be employed. Three are used in some achromatic telescopes; a greater number would diminish the light too much by their successive reflections; besides, when the achromatic effect of these instruments is determined with care, by the methods already pointed out, their imperfections are no longer the consequence of unequal refrangibility in the rays of light, but of the diffusion of the foci, in which spherical lenses, having any sensible aperture, concentrate the several incident pencils, which cover their whole surface.

185. Finally, the absolute quantity of dispersion, measured between two determinate colours, is as variable as the law itself, according to which the different rays arrange themselves in each spectrum. This is plainly indicated in the above process, by the varieties of coloured fringes obtained when two different substances are compensated by each other.

If substances are compared together, which have very unequal refracting powers, the dispersive powers will, in general, be found to be so likewise, and in the same direction, the most refractive substances being the most dispersive. For example,

the oxide of lead introduced into the composition of glass, augments to a considerable degree, its dispersive power; it augments likewise its refractive power, although in a smaller proportion. Of all the substances which were tried by M. Cauchoix and myself, the liquid known in chemistry under the name of sulphuret of carbon, appeared to have the greatest dispersive power. The dispersion produced by it was ten times that of water, under similar circumstances; the refractive powers of sulphur, and of solid carbon are also very considerable. Still this correspondence between the increments of refraction and dispersion is very far from being general, especially when the ratios of refraction differ but little. The essential oils of citron and turpentine, muriatic acid, pure or saturated with ammonio-muriate of mercury, disperse more than crown glass, but refract less, as has been well ascertained. The same is true in a number of other instances; so that the relation of the dispersive forces of bodies to their chemical composition, is even more difficult to be foreseen than that of the refractive forces.

John Dollond, a celebrated English optician, was the first who proved, by experiment, the error of Newton, respecting the possibility of obtaining an achromatic compensation, preserving at the same time an excess of refraction. Euler supposed this to be possible, because it was realized, at least very nearly, in the construction of the eye, which is achromatic when well adjusted, and sensibly unites all the refracted rays in the image upon the retina, and paints the objects in their proper colours, as may be seen by taking the eye of an animal recently killed, and removing the outer coating from the back part, and examining the images formed there. But this remark, upon an organ thus composed, was not sufficient to develope the true principles upon which the compensation is founded. Euler proposed several hypothetical laws, which might produce this effect. It was in trying these laws that Dollond was lead to repeat some of the experiments of Newton, upon compensation by prisms of different refracting substances; and being led by chance, or a happy conjecture, to try some of a nature very different from each other, like crown glass and flint glass, he discovered that the refraction produced by crown glass remained predominant, when the angles of the prisms were such that the compensation was sensibly complete. By preserving the same relation between the prismatic

borders of concave and convex lenses, made of these two substances, Dollond obtained achromatic object-glasses, which enabled him greatly to enlarge the apertures hitherto employed. Although great use has been made of this discovery, but little pains have been taken to perfect it, and opticians are still compelled to use the rules of compensation given by Dollond, even in instances where they are no longer applicable; and when under the necessity of deviating from them, on account of great difference in the substances, they have resorted to expensive and imperfect trials. These are the considerations which led M. Cauchoix and myself to seek an exact process, such as has here been given.

Of the Dispersion which accompanies Extraordinary Refraction, and the Separation of the Axes of Double Refraction with respect to the different Simple Rays.

186. It has been mentioned, that in crystals endued with double refraction, the extraordinary image is generally coloured, as well as the ordinary. This phenomenon, therefore, proves that, at equal incidences, the extraordinary velocities are unequal for the different simple rays, as analogy alone would indicate; but it may be easily shown that the mode of dispersion resulting from it must generally be more complicated than that for images where the velocity is constant. Indeed, confining ourselves, for the sake of simplicity, to crystals of one axis, if we call the ordinary velocity v, and the angle formed by the extraordinary ray with the single axis U; we have seen that the extraordinary velocity v', is given by the formula,

$$v'^2 = v^2 + k \sin^2 U,$$

in which k is a constant coefficient peculiar to each crystal, and which is positive in some and negative in others. Now, experiments show, that generally the values of this coefficient are sensibly different for the different simple rays, and increase with the refrangibility. Their variations, or rather those of the product $k \sin^2 U$, which result from them, combine therefore with those of the square of the ordinary velocity to form v', and as the values of v increase also with the refrangibility of the rays,

it is manifest that if k is positive, as is the case in attractive crystals, the two causes of variation conspire in the extraordinary velocity; while on the contrary, if k is negative, as it in fact is in repulsive crystals, they counteract each other. In this last case, therefore, it is possible that they may nearly or entirely destroy each other; and then the extraordinary image may be sensibly white, although very powerfully refracted. Indeed, this singular phenomenon, is exhibited in the rhomboids of Iceland spar, when the incident ray SI, is directed towards the small solid Fig. 78.
angle B'; because then the ordinary refraction tends to disperse the spectrum, by causing the most refrangible rays to approach the normal to the point of incidence; while the repulsive force, emanating from the axis IA', tends to disperse them in the opposite direction. Malus found that these two opposite causes counterbalance each other when the incidence is about 40°. Then the ordinary image only is dispersed; and the extraordinary image, although strongly refracted, is sensibly white.

187. It has likewise been remarked that, according to a very 118.
ingenious discovery made by the son of the celebrated Sir William Herschel, the axes of double refraction in crystals, are not always the same for all kinds of simple rays, but have different positions, and different inclinations to each other, for these different rays. This dispersion has as yet been observed only in crystals of two axes. Indeed, it is easy to see that it is not possible in others, simply from the condition of symmetry of faces, which is necessary for the existence of a single axis. For the 120.
same reason, in crystals with dispersed axes, all the pairs of axes are comprehended in the same plane, and have the same intermediate line common. But the direction of the dispersion is subject to no rule. In some crystals, such as the sulphate of barytes, the nitrate of potash, aragonite, sugar, and hyposulphate of strontian, the axes of the red rays are less inclined to each other than those which correspond to the violet rays. The reverse takes place in borax, siberian mica, sulphate of magnesia, white topaz, and tartrate of potash and soda. The phenomenon is particularly excessive in the last salt. According to Mr Herschel, the inclination of the axes in this, is 55° 14′ for the violet rays, and 75° 42′ for the red rays, each being taken at the extremity of the spectrum. It is obvious that these differences as to position and angles must affect the course of the refracted rays, calculated

116. according to the general law, and therefore in crystals where they are sensible, we are obliged to specify the particular species of rays to which the formulas are to be applied. These phenomena must evidently render the laws of dispersion very complicated, in the crystals under consideration. But it was not in this way that Mr Herschel made the discovery of the separation of the axes. The tartrate of potash and soda, where it is so considerable, has a double refraction, so very feeble, that it is hardly capable of being measured; and in crystals hitherto observed, where the double refraction is most powerful, the axes are generally very little dispersed. But the phenomenon of their dispersion is rendered very evident, independently of the doubling of the images, by certain phenomena of colour to be explained hereafter; and it was by these that Mr Herschel was led to the discovery.

Dioptric Instruments consisting of several Glasses.

188. The most complicated dioptric instruments may be considered as consisting essentially of two glasses. The first, called the *object-glass*, receives the light immediately from the object, and forms an image of it at its focus. The second is called the *eye-glass*, and is placed near the eye for the purpose of viewing this image, which, according to the relative focal distance of the two glasses, and the position in which they are placed, will appear erect or inverted, magnified or diminished. This system may be greatly improved by making the eye lens to consist of several glasses, properly disposed, and rendering the object lens achromatic when it is possible. By this means greater distinctness and a higher power may be obtained. All the varieties, however, may be reduced to the same principle. Whatever be the number and curvature of the glasses, they must all have their axes in the same straight line and be firmly fixed in a tube, consisting of several pieces that slide within each other, for the purpose of varying the distance of the eye-glass from the object-glass. This tube should be blackened on the inside for the purpose of absorbing all the light which strikes upon its sides; for

no rays but those which come nearly in the direction of the axis, common to all the lenses, can be of use in vision. Hence, in order to insulate these rays completely, a number of transverse partitions with circular openings, called *diaphragms*, are placed in the interior of the tube, being coloured black, in order to arrest by their opacity the rays which are too oblique.

In general, all instruments of this kind may properly be considered as cameræ obscuræ, or dark rooms, of small dimensions and a field of view of little extent. This last limitation is required in order to the enlargement to which it is to be subjected in the image; for it would appear distorted if it were not reduced to very small dimensions.

189. Each kind of instrument is appropriated to a particular purpose; some are employed in examining very minute objects at a short distance by enlarging their image; these are called *microscopes;* others are used in viewing distant objects, under a greater angle than they present when seen by the naked eye, they being seen with equal distinctness at the same time; these are called *telescopes*. In both these kinds of instruments, the same principles are employed; and they are adapted to their respective objects by introducing some peculiarities in the construction. This will be seen as we proceed to examine successively those which are the most common and most useful.

Compound Microscope.

190. THE object-glass of this instrument is a small lens A_1 of a very short focus, before which are placed minute objects $S\Sigma$, at a distance A_1P or Δ, which exceeds very little the distance A_1F_1 of the principle focus. Behind this lens is formed an inverted image $f_1\,\varphi_1$, at a distance A_1P_1 or S, much greater than Δ; and if the magnitude of $S\Sigma$ is expressed by I, that of $f_1\,\varphi_1$, will be expressed by $\frac{IS}{\Delta}$; hence this image is likewise much greater than the object $S\Sigma$. If this degree of magnifying power were sufficient, the image might be received upon ground glass, placed at $f_1\,\varphi_1$, and viewed in this situation by the naked eye. But it is evident, that the effect would be increased still more if the Fig. 121.

eye were furnished with a magnifier A_2, placed in such a manner that the image $f_1 \varphi_1$, may be a little within the principal focal distance; for there would result a second image $f_2 \varphi_2$, likewise inverted, but greater than $f_1 \varphi_1$. There will then be no need of a ground glass, which would absorb a great portion of the light; for the particular foci, which united, form the image $f_1 \varphi_1$, will have with respect to the lens the same effect as so many radiant points, only much more luminous. Such is the manner in which the eye-glass A_2 is disposed in the compound microscope. The two lenses are fixed in the ends of a tube, the parts of which slide in each other for the purpose of varying the interval which separates the glasses.

191. Here, as in simple microscopes, the magnifying power is in the ratio of the absolute dimensions of the object $S \Sigma$ to those of the last image $f_2 \varphi_2$. Hence, other things being the same, it magnifies objects the more according as the focal distances of the lenses are less. The focal distance $A_1 F_1$ of the object-glass being diminished, the image $f_1 \varphi_1$, formed behind this glass, becomes greater at the same distance. By shortening the focal distance $A_2 F_2$ of the eye-glass, the image $f_2 \varphi_2$, resulting from $f_1 \varphi_1$, is enlarged, the distance of distinct vision being the same. The first kind of variation is limited by the difficulty of accurately constructing very small lenses, the injurious effect of increasing the angles of incidence and emergence of the rays at the surfaces of these lenses, and lastly, the very small quantity of light which they are capable of receiving and transmitting. But the increase of power derived from contracting the eye-glass, is still more limited, on account of the necessity of having it of considerable magnitude. Indeed, the surface of the object-glass being small, the pencil of refracted light that comes from any part of the object, is very minute; so that when it is concentrated in one of the points of the image $f_1 \varphi_1$, and radiates anew from this point, it will not cover the whole of the eye-glass A_2, but will fall upon only a small portion of it; hence, if the eye-glass is so contracted in its dimensions that the pencil from a given point of the image shall not meet this glass, it will be wholly lost to the eye. The *field of view* of the instrument, that is, the space embraced by the eye as it looks through the lenses that compose it, will be limited also by the last rays which meet the extreme border of the eye-glass; this is another reason why it cannot be indefinitely diminished.

• 192. From the smallness of the refracted pencils the field of view may be considered as limited by the pencils whose axes $A_1 f_1$, $A_1 \varphi_1$, graze the borders of the glass. No notice need now be taken of the more oblique pencils, of which only a part falls upon the eye-glass, on account of the feebleness of the light; they are moreover excluded by a diaphragm, placed between the glasses, where the image $f_1 \varphi_1$ is formed. Hence, if SA_1, ΣA_1, represent the axes of the extreme pencils, the extent of the field of view will be equal to the angle $SA_1\Sigma$, or $I_2 A_1 i_2$; it may then be easily calculated; for if we call h the interval $A_1 A_2$ between the two lenses, and z_2 the semidiameter $I_2 A_2$ of the second, half of this angle, or $I_2 A_1 A_2$, will have for its trigonometrical tangent the ratio $\frac{z^2}{h}$; from this expression the angle may be obtained by the tables. In order that the eye may embrace the whole of this extent, it must be placed at the point O of the axis, where are collected all the extreme emerging rays, and generally all the most minute pencils which proceed from the several points of the last image $f_2 \varphi_2$. Hence the distance of this image from the point O should be equal to the distance at which distinct vision takes place. Each observer can fulfil this condition by varying, according to the reach of his eye, the interval h between the two lenses, or the distance Δ of the object, which varies also the magnifying power and the extent of the field of view. The eye being thus placed at the proper point O, will perceive the whole of the field of view, and would perceive it even if the opening of the pupil were infinitely small. But when removed only a small distance from this point, it would scarcely receive any rays, except those in the axis, and the field would vanish, were it not for the sensible opening of the pupil, which enables the eye to take in the whole field when it is not situated exactly at the point O.

193. By introducing the several particulars of this construction into the general formulas which express the course of the rays of light through several spherical lenses, arranged on the axis, we obtain the exact measure of all the phenomena above indicated. But they may be obtained also by experiment.

The extent of the field of view is determined immediately by tracing the limits of the space visible at a given distance $A_1 P$. The magnifying power requires certain details which we proceed to make known.

Fig. 122. The body of a compound microscope generally consists of three tubes that slide within each other. The upper part *GE*, in which the eye glass is fixed, and which for this reason is called the eye piece, is fitted to slide firmly in the tube *FC*; this in its turn slides firmly, by means of friction, in the lower tube *BA*, in the bottom of which the object-glass is screwed. This being premised, the eye piece *GE* is taken out, and at *DD* a circular diaphragm is introduced, its diameter having been previously measured; then, replacing the eye-piece, it is made to slide down till the borders of the diaphragm are distinctly seen, which will then be found precisely at the point where the image of an object, formed by the object-glass, must be brought, in order to be distinctly seen through the eye-glass. The diaphragm *DD* is usually in the piece *FC*; and the proper distance between the eye-glass and the diaphragm, for different eyes, is obtained by drawing out or pushing in the piece *GE*.

Before the object-glass *A*, there is a double circular ring of metal *SS*, in which are placed the objects which are to be examined with the microscope; or rather these objects are fixed on plates of glass or mica, which are made to slide between the rings. The instrument is mounted in such a manner that the rings *SS* may be made to approach to, or recede from, the object lens, for the purpose of placing the object at a proper distance from this lens. Then, when the magnifying power is to be measured, instead of an object, a piece of glass is made to slide in the rings. On this glass are drawn, with a diamond, a number of parallel lines, the distances of which from each other are accurately known; let them be thousandths of an inch, for example. A glass thus divided is called a *micrometer*. When this is placed, the middle piece *FC* is let down to a certain fixed point. Then, without touching the eye-glass, the whole body of the microscope *GFC* is raised or lowered till the lines on the glass micrometer are distinctly seen. The image will thus be found to be exactly in the diaphragm *DD*, since it is only in this situation that objects can be distinctly seen through the eye-glass. The divisions, contained in the diaphragm, are thus counted; let us suppose the number to be equal to m, and that the real diameter of the diaphragm is M; it follows therefore that m divisions of the micrometer, thus magnified, are equal in extent to M. Hence the magnifying power of the object-

glass will be represented by $\frac{M}{m}$. Suppose the micrometer to be divided into thousandhts of an inch, and the diameter of the diaphragm into 5 tenths of an inch. The instrument being brought to the proper point of view, I will suppose that 20 parallel divisions are contained in the diameter DD; there are then 20 thousandths of an inch, which, by the power of the object-glass, become equal to 5 tenths, or 500 thousandths of an inch. Thus the magnifying power produced by the object-glass, at this distance, is found to be $\frac{500}{20}$ or 25; hence we conclude, in general, that the first image $f_1 \varphi_1$, formed behind the object-glass, is twenty-five times as large as the object.

194. Now the enlargement of this image $f_1 \varphi_1$, produced by the object-glass, may be calculated upon the principles made use of in the case of a simple magnifying glass, from its focal distance and the distance of distinct vision, the rule heretofore given 62.
being reversed. Thus the total enlargement produced by the compound microscope, will be the product of these two partial enlargements. If, for example, in the situation where the object is placed, it is magnified twenty-five times by the object-glass alone, and its image is magnified ten times by the eye-glass, the total effect will be equal to the product of 25 by 10 or 250.

The proportions thus determined belong to the distance Δ, at Fig 121.
which we place the object from the object-glass. The effect is increased if we bring it nearer the principal focus F, and diminished if we remove it farther off; for, in the first case, the image $f_1 \varphi_1$, is thrown further from the object-glass A_1, and, in the second, it is brought nearer. Accordingly, if we do not wish to alter the eye-glass, it would be necessary to move the intermediate piece FC, in order to lengthen or shorten the tube, in such a manner that the new image may always be formed within the borders of the diaphragm DD, and thus be found at the constant distance from the eye-glass which is adapted to distinct vision. We might repeat, in each of these new positions, the process for determining the enlargement produced by the object-glass, but this is not necessary; for it is shown by calculation, that, for the same distance of distinct vision, the values of total enlargement are sensibly proportional to the distance A_1E, or the interval between the object-glass and the eye-glass, diminished by the focal distance of the eye-glass. It is sufficient then

to measure these distances in the first experiment, when the intermediate piece *FC*, is pushed down to its limit; and by tracing on the tube a scale of equal parts, which will indicate, for all other cases, the quantity by which it is lengthened, we can deduce the magnifying power of the microscope, by the proportion we have stated. Or, if we wish, we can mark these enlargements on the tube itself, against a certain number of divisions, sufficiently near together, to allow the intermediate ones to be calculated by simple means. But these values, being compounded of the effect of the object-glass and that of the eye-glass, will still vary for different eyes, according to the distance of distinct vision; because it will be necessary, in order to adapt the distance of distinct vision to different eyes, to increase or diminish the distance of the eye-glass from the diaphragm; so that the same instrument will always magnify more for longsighted than for shortsighted persons, as we have shown in the case of simple microscopes.

195. When the microscope is once prepared in the manner above described, it may be employed for the purpose of obtaining the absolute dimensions of small objects, and the result will not be far from the truth. We might place one of these objects immediately on the glass micrometer, and see with the microscope how many of its divisions are covered by it, were it not that the thickness of the object, however small we suppose it, has an important influence on the place of the image, on account of its proximity to the principal focus of the object-glass; so that we can never see it distinctly through the eye-glass, and the divisions of the micrometer at the same time. To remedy this inconvenience, we place another micrometer in the interior of the microscope, exactly in the place where the first image is formed, that is, on the diaphragm *DD*, which answers to the focus of the eye-glass. Then we easily observe *how many divisions the image of the object takes up on this interior micrometer.* This number, divided by that which expresses the enlargement produced by the object-glass alone, for its actual distance from the image $f_1\ \varphi_1$, or the micrometer, gives the absolute size of the object.

196. The above method was long since made use of by M. Charles. The limits of this treatise will not allow a detail of all the improvements introduced by this skillful observer. But I

will mention some particulars to be attended to without which the microscope would be altogether useless.

In the first place, it is absolutely necessary to illuminate strongly the objects we wish to observe. These objects being hardly ever luminous of themselves, emit directly very few rays; and only a small number of these are admitted into the microscope, on account of the very small opening which must necessarily be given to the object-lens. If then we confine ourselves to this small degree of light, the image will be so faint, that we can hardly ever perceive it, however little it may be magnified. On this account we illuminate the object strongly, by throwing upon it, from a slightly concave mirror, the ordinary light of the sky, or that of a lamp condensed by means of a converging glass. If the object is opaque, we illuminate it in this manner by light from above; but if it is transparent, we generally receive the light from below. I say, generally, because there are some cases in which it is best to have the light directed otherwise. When, for instance, we wish to observe the divisions of the micrometer, in order to determine the magnifying power, we never see them better than by an oblique reflection; then they are delineated in black on the glass plate. There are some of these micrometers which contain 900 visible marks in the space of a French line or $\frac{8}{90}$ of an English inch.

197. Another indispensable precaution is to place diaphragms in the interior of the instrument, to limit the field of view, and to exclude all that part of the image which is ill defined. For in all the preceding considerations, we have supposed the incidences and emergences infinitely small. They are not so in reality, and they are the less so, according as we give to lenses a greater opening. Hence the concentration of the rays in a single focus, the regular formation of the image, its perfect similarity to the object, and all the other properties which exist under very small inclinations, are only approximations from which we deviate more and more, in proportion as the glass used has a greater aperture. Now we must deviate only just so much as to prevent vision from becoming too defective; and this we do, by limiting the field of view by diaphragms, differing in extent according as the case may require; and a very simple method, is to retrench from the image whatever might affect the distinctness of its outlines.

198. Dr Brewster has contrived a very ingenious method of employing the microscope for measuring the ratios of refraction in liquids, and a great number of substances partially solid, such as wax, gum elastic, &c., which are opaque in the mass, but which become translucid, or even transparent, when made sufficiently thin. For this purpose, we fix before the object lens, and nearly in contact with it, a very thin plate of glass with parallel faces; then we bring the microscope to the point of distance proper for seeing distinctly, through this system, the marks of the object-glass micrometer. This being done, we insert between the glass plate and the lens, a drop of the liquid whose refraction we wish to examine; or if it is a solid substance, we detach a very thin lamina and press it strongly between the glass plate and the lens, until it is moulded, as it were, upon the surface of the lens. This operation forms, with the substance pressed, a true divergent meniscus, whose anterior surface is plane, and whose posterior surface has the same curvature as the anterior surface of the object lens. The interposition of this meniscus necessarily increases the distance at which the image of the object is formed behind the object lens; so that if we keep the object-glass micrometer at the same anterior distance, we must lengthen the body of the microscope in order to see clearly the image in the diaphragm, or what leads to the same result, we may leave the body of the microscope unaltered, and increase the distance of the object from the lens. The amount of this alteration depends upon the curvature of the object-lens and upon the nature of the substance interposed. If the curvature is known, we deduce from this amount, the ratio of refraction of the substance. If it is unknown, and it is always best to make this supposition, then we begin by forming the meniscus with pure water, whose ratio of refraction is known, being, according to Newton, equal to $\frac{529}{396}$ or 1,33586; and by comparing the alteration of distance necessary in this case, with that required for the particular substance afterwards subjected to experiment, we deduce the ratio of refraction of this substance compared with that of water. The application of this process requires much skill and address, because the accuracy of it depends upon the more or less perfect exactness with which we bring the image into the diaphragm, in doing which, our only guide is the condition of greatest distinctness of vision.

Still it admits of many very useful applications, both for substances which only become transparent when reduced to their laminæ, and for liquids of which we possess only very small quantities, a single drop being sufficient for the observation.

Amplifying Glass and Achromatic Eye-Glasses.

199. THE imperfections of the microscope arise in a great degree from the want of achromatic lenses, which is the more felt according as we attempt to make use of higher degrees of magnifing power. Unfortunately it is impossible to remedy this defect entirely, since it is in vain to think of forming achromatic lenses so small as those which the microscope requires. But this evil may be, to a considerable degree, removed by a method which suggested itself to practical men, before any theory had been devised to explain its effect, or to show how it might be employed in the most advantageous manner. This method consists in placing, in the interior of the microscope, behind or before the first image $f_1\ \varphi_1$, a third converging glass, of a focus properly determined. Then the course of the rays will be such as is represented in figures 123, 124. The first disposition, figure 123, was invented by Campani; the other, figure 124, by Ramsden.

The employment of this glass, called the *amplifying glass*, is common to all dioptric instruments. Its evident use is so to collect the pencils separated by the object-glass, to concentrate them in a smaller space, and thus to render the image more distinct, smaller, and consequently to make a greater part of the object visible by a given eye-glass. But there is another less obvious use, which consists in the achromatic influence which this glass exerts.

200. When the rays coming from an object, have been refracted by any system whatever of spherical lenses, which form images of it about the axis AX, owing to the unequal refrangi- Fig. 125.
bility of light, the foci of the rays of different colours are not, in general, formed at the same distance; so that if φ_1 is the focus of the violet rays, φ_2 will be that of the indigo, φ_3 that of the blue, and finally φ_7 that of the red; and as the same property

belongs to the radiating points situated without the axis, there will be formed in φ_1, a violet image VV, in φ_2 a blue image BB, and in φ_7 a red image RR; and the same cause which distributes them at different distances will give them also different dimensions. Now, if the eye be placed at some point O, in the axis AX, for the purpose of looking at these images, it will, in the first place, suffer the inconvenience of their unequal distance, which will prevent them from being seen together at the exact distance necessary for distinct vision. Moreover, it will be disagreebly affected by the inequality in their size; for, as they project beyond one another, they will exhibit the outlines of objects bordered with coloured fringes of red, violet, or the intermediate colours, according as one or the other may predominate, in the course of the refractions they severally undergo. A great advantage, therefore, will be obtained if we can regulate the size of these images, in such a manner as to make them exactly proportional to their distances from the eye, as represented in figure 126; for then the eye seeing all their borders in the same straight line VRO, will receive at once, from these borders, the sensation of all these species of rays, and consequently the coloured fringes will disappear. Now this disposition, which would seem at first to be very complicated and difficult, is found in fact to be extremely simple; and is precisely the effect produced by the amplifying glass, when its focal distance, and its position with respect to the other glasses, are properly determined. This can be done only by calculation, and consequently, I cannot here state the conditions. I will simply remark, that this arrangement, in order to be possible, requires that there should be in the instrument, at least two glasses A_2, A_3, besides the object-glass A_1; for, with the object-glass and only one other, we should not be able to render the borders of the last image colourless, except in one particular situation of the object. Accordingly, in all the applications that follow, *the achromatic eye-glass must always be composed of at least two lenses.*

Fig. 123. 201. The achromatic eye-glass of Campani, is the one always employed in compound microscopes, and generally, in instruments where we do not wish to extend fixed wires over the image given by the object-glass. But when these wires become necessary, as in astronomical instruments destined for measuring, where we are obliged to fix precisely the direction of

the rays which come from the heavenly body to the eye, at a known instant, we cannot employ this arrangement; because, in drawing out or pushing in the eye-glass to accommodate it to different eyes, we should necessarily move the wires, and if this movement were not exactly in the axis of the telescope (and we cannot suppose it would be), the successive passages of the body would not be comparable with each other. In this case, the eye-glass of Ramsden is particularly applicable; because, Fig. 124. being situated completely beyond the first image $f_1\ \varphi_1$ it can be drawn out or pushed in, without moving the wires extended at the place where the image is formed. Accordingly, we always employ it under these circumstances, and it was for this purpose that that celebrated artist contrived it. By examining the effect of the eye-glass of Campani, in microscopes, M. Cauchoix found it advantageous to give to the amplifying glass the form of a meniscus convex towards the object-glass. As to the enlargement produced by this apparatus, we shall explain hereafter the means of determining it according to a process devised by M. Arago, and which is applicable to all optical instruments. We repeat this observation for two different distances of the object from the object-glass; which will cause the image to advance or recede, and make it necessary to move the two lenses of the compound eye-glass, in order to bring it to the true point for distinct vision. We shall thus have two known enlargements, for a known lengthening of the tube; their difference, distributed uniformly among all the intermediate lengthenings, will give the corresponding intermediate enlargements, which may be traced on the tube.

A bare inspection of figures 123, 124, will show that these eye-glasses do not change the inversion of the objects produced by the object-glass; but their erect position may be restored by employing more than two glasses, as we shall see hereafter. This is done, in certain cases, in the telescope, which is next to be considered.

Dioptric or Refracting Telescope.

202. If we enlarge the object-glass of the microscope and remove the object to a great distance, we shall have a refracting telescope, which, like the microscope, may be composed of two, three, or a greater number of glasses. But the first lens A_1, being no longer of very small dimensions, may be formed of an achromatic assemblage of several glasses in contact with each other, which will give the same focal distance for all the coloured images produced by it. These images will, in reality, be separated from each other, in traversing the eye-glasses to arrive at the eye. But besides that this separation will be very small, in consequence of the short distance through which they move, the effect will become altogether insensible, if the eye-glasses are combined in conformity to the principles laid down in a preceding section; for then, the coloured images which are presented to the eye, being very close to each other, will, at the same time, have dimensions proportional to their distances, so that the achromatic effect will appear perfect. Accordingly, this arrangement is generally practised.

203. The first and most simple kind of telescope is that called *astronomical*. It is represented in figure 127. The object-glass A_1, is a converging lens, and it must always be such, in order to throw the image on the posterior side towards the eye. The eye-glass A_2 is also a converging lens, and the last image $f_2\,\varphi_2$ is inverted.

This disposition is exactly similar to that of the microscope with two glasses, except with respect to the diameter of the object-glass A_1. Hence result larger pencils and a more consider-
Fig. 121. able accumulation of light. But if we consider the axes of these pencils, which enter through the centre of the object glass, their course is absolutely the same, and accordingly the conditions which they must fulfil, in order to produce in the eye distinct vision, will be the same as before; only, as the object is now very remote from the object-glass, the image is formed behind it at nearly the same invariable distance, which is that of its principal focus. Moreover, since its distance from us is so great that we cannot have a distinct idea of it, while the

distance of the last image from the eye is so small as to admit of no comparison with the other, the enlargement is no longer measured by the actual ratio that exists between the size of the object and that of its image, but by the ratio of the visual angles $SA_1\Sigma, f_2 O \varphi_2$, which the object and the image respectively subtend at the eye. If the instrument is composed of only two glasses, this ratio is sensibly equal to the focal distance of the object-glass, divided by the focal distance of the eye-glass, at least when this last may be considered as very small compared with the distance of distinct vision. Such accordingly will be the value of the magnifying power. But here, as in the microscope, we hardly ever employ the simple eye-glass, on account of the colour which it produces on the borders of the last image $f_2 \varphi_2$, even when the object glass is achromatic. If the instrument is intended for astronomical observations, where distinctness of image and abundance of light, are the only essential conditions, we employ the compound eye-glass of Ramsden, or that of Campani. These do not give the image erect; but this is not important in astronomical observations.

204. The same cannot be said with respect to telescopes intended for terrestrial objects. Here it is essential that the last image, situated near the eye, should represent objects erect. This purpose is effected by making the eye-glass to consist of four separate lenses, of which the two first A_2, A_3, nearest the Fig. 123. object-glass A_1, are intended simply to render the image erect, while the two last A_4, A_5, situated near the eye, complete the achromatic effect upon the borders, and have also the same ratio to each other, as those in the eye-glass of Ramsden or Campani. The magnifying power of the telescope depends upon the foci of these five glasses and their intervals asunder. Now, if we give to the two last, situated next the eye, the distances which are necessary for rendering the borders colourless, we can still vary the positions of the rest within certain limits, without the instrument failing to perform well. But the magnifying power will vary, and we can, by this single movement, cause it to pass through all its degrees. M. Cauchoix has effected this in his terrestrial telescopes, which he has called for this reason *polyalde*. They thus give at pleasure, a feeble or a great magnifying power, which is often very advantageous, the first being more convenient in case of a fog, and the second in clear weather.

Telescopes thus disposed and intended to be portable, vary their magnifying power from twenty to forty times, or from thirty to fifty times. The magnifying power of the greatest astronomical telescope hitherto known, amo unts to 1000 or 1200 times, this estimate, like the preceding, being applied to the diameters of the objects. But we do not adapt to these instruments the polyalde apparatus, because the multiplicity of reflections from the glasses would weaken the light too much; and when we wish to alter the magnifying power, we do it by changing the eye-glass.

205. There are also telescopes in which the object-glass, always converging, is combined with a simple eye-glass, but Fig. 129. diverging. This arrangement, invented by Galileo, is still made use of in opera-glasses. It causes the objects to be seen erect.

In this case, the first image $f_1\ \varphi_1$, given by the object-glass A_1, is not actually formed, although in estimating the results, we must consider it as really existing. Before the focus P_1, where it ought to be produced, we place the diverging eye-glass A_2, at such a distance that the convergent pencils, tending towards the points f_1, φ_1, may be changed to divergent pencils, departing from other points f_2, φ_2, situated before the eye-glass, and at a distance equal to that of distinct vision. These points form then the last image, or that which the eye perceives. The deviation produced by the eye-glass in the axes of the pencils which compose this image, makes it erect, because their directions cut each other before reaching the eye. But for this very reason, the eye cannot be placed at the point of meeting O, which falls in the interior of the tube; but, foregoing this favourable position, it must be placed somewhere without, in the axis of the glasses, at O', for example, where it receives only the divergent portion of each pencil, which passes, in this place, sufficiently near the axis A_1X, to be able to enter the pupil of the eye. It follows from this disposition, that in proportion as the eye is removed from the point of meeting O, a greater number of pencils, diverging beyond the space embraced by the pupil, escape it entirely; and as this disappearance necessarily commences with those which are most distant from the axis, and which form the borders of the image, it follows that the field of view is diminished in proportion as the eye is removed. Accordingly, the position of the eye which is the nearest possible to the eye-glass, is that

which gives the greatest field of view. Notwithstanding these inconveniences, the use of diverging eye lenses in opera-glasses has two advantages; one is, that it makes the object appear erect; and the other, that it shortens the total length of the instrument, by being placed within the focus of the object-glass, whereas converging eye-glasses lengthen it, by being placed without. We accordingly employ in these glasses, only a simple eye-lens, although it inevitably produces colour, even when the object-glass is achromatic; because, being intended to be used in the evening, in places where the degree of illumination is less than daylight, the colours which are developed, when the instrument is well made, are not very vivid, especially if we take care to place the pupil in the axis; we should, moreover, weaken the light too much, if we made use of eye-glasses composed of several lenses.

Instruments which consist of a Combination of Mirrors and Spherical Lenses.

206. Catoptric instruments of whatever kind consist of mirrors either concave or convex, arranged in such a manner as to give, by reflection, distinct images of objects, which are observed by means of a simple or compound eye-glass. Both telescopes and microscopes may be made in this way; but we shall here consider only the former, the latter being no longer in use; and we shall confine ourselves to the illustration of the most common case, in which the distance of the object may be considered as infinite. It will easily be perceived that the method must be the same in all other cases.

The simplest of all reflecting telescopes is that which is represented in figure 130. It is formed with a single concave mirror *MM*, which, receiving the rays from a distant object $S\Sigma$, forms with them, at its focus, an image $f_1\ \varphi_1$, which we look at through an eye-glass for the purpose of magnifying it. But in this case, the observer being interposed between the mirror and the object, necessarily interrupts a part of the incident rays; on this account, we cannot employ such an arrangement, except with very large mirrors; and to prevent as much as possible the loss

of light, we direct the axis of the instrument a little obliquely with respect to the object, in order that the image may be formed without the axis, and the top of the head only intervene. Then, if the mirror be very large, the loss of light will be small, compared with that which more multiplied reflections and refractions would occasion. Sir William Herschel constructed, in this manner, a large telescope of forty feet focus, with which he made a number of discoveries. There was a similar one at the observatory of Lilienthal, in the hands of M. Schroeter. The trouble of moving such large instruments makes it very difficult to use them.

207. Next to the form above described, the most simple is the one invented by Newton. It consists in placing in the interior of the telescope, and near its principal focus F_1, a small plane mirror *m m*, inclined to the axis at an angle of 45°, and of dimensions just sufficient to receive all the reflected rays. This mirror changes the direction of the rays, but in changing them, does not at all alter their convergence. It only throws the focus in a direction perpendicular to the axis, and at the same distance from the point where the mirror cuts the axis, at which it would be if formed in its prolongation. Opposite to this new direction, we make a lateral opening in the tube of the telescope, in order that the rays may emerge, and we look at the image through a simple or compound eye-glass. This arrangement prevents the direct interposition of the observer, and allows us to employ mirrors of all dimensions. But it occasions a considerable loss of light by the second reflection which it requires, especially since the reflection takes place from a metallic surface, whose absorbing power is always very great. Accordingly, in telescopes of this kind constructed by Newton himself, in order to change the direction of the image, he made use of total interior reflection, at the second surface of a rectangular glass prism, so situated in the telescope, that one of the sides containing the right angle, should be perpendicular to the axis of the telescope, as represented in figure 132.

Fig. 131.

208. The position of the observer at the side of the telescope is inconvenient, when he is seeking, with the telescope, any of the heavenly bodies. It is much better when in the direction of the axis of the instrument. This purpose is effected by the construction of Gregory, represented in figure 133. It consists in substi-

tuting for the plane mirror, a small concave mirror *m m*, which reflects the rays coming from the large mirror, and sends them towards its centre, where an opening is made for their passage. Then a second image of the object is formed behind this opening, at the compound focus of the two mirrors, which is viewed through an eye-glass placed in the axis. If we suppose the incident rays parallel, the first image will be formed at F_1, the principal focus of the large mirror, and will take the place of an object with respect to the second. Accordingly, from what has been laid 15. down, it follows that the first image must be situated between the centre of curvature and the principal focus of the second mirror, in order that the second image may be thrown beyond the first, towards the observer.

209. Cassegrain further modified this construction, by substituting in place of the small concave mirror, a small convex mirror *m m*, in order that the aberrations of sphericity produced Fig. 134. by the two mirrors, might mutually compensate each other. In this case, that the second image may be formed on the side of the observer, it is necessary that the first should not be actually formed; but that its imaginary place should fall beyond the small mirror, between its surface and the principal focus. This result which was not considered in article 15, because it supposes the incident rays convergent towards the mirror, is easily demonstrated by the general method of article 13.

210. In figures 130, 131, 133, and 134, for the sake of simplicity, we have only represented a simple eye-glass. But, it is in fact necessary, in order to render the last image colourless, that the eye-glass should consist of two lenses, arranged according to the principles laid down in articles 200, 201. We have also been obliged, in the figures, to increase very much the true dimensions of the eye-glass, compared with those of the mirror *MM*, that we might be able to represent the mass of rays and succession of images, in a conspicuous manner.

It is hardly necessary to remark, that in these telescopes, the mirrors are firmly fixed in the axis of a tube, sufficiently long to permit only those rays which are nearly perpendicular, to fall upon them. Indeed, we often contract this opening by means of diaphragms, for the express purpose of interrupting those rays which would fall on the borders of the mirror, this being never so well executed as the centre. The tubes ought to be blackened on

the inside, like those of refracting telescopes, for the better absorption of the light irregularly reflected from their sides. In fine, they must be mounted in such a manner that they may be directed at pleasure, towards the different points of space.

Method of M. Arago for determining the Magnifying Power of Optical Instruments.

211. It has already been observed, that in instruments intended for viewing distant objects, the magnifying power is equal to the ratio of the visual angles, under which the same object is seen with the naked eye, and through the system of glasses of which the instrument is composed. If the object is sufficiently near the eye to allow its distance, in these two cases, to be compared, it is necessary to combine the ratio of the visual angles with the ratio of the real and apparent distances of the object, in order to deduce the ratio of its real and apparent magnitudes, both taken at the distance of distinct vision.

The process of M. Arago gives immediately the ratio of the visual angles. For this purpose, we take a double prism of rock crystal, similar to those whose construction is represented in figure 92, and which serve for double-image micrometers. We measure the angle $O\,c\,E$ or C, at which it divides the light. This can be done in a very simple manner, as we shall soon see. We next place the double prism behind the eye-glass of the instrument under examination, which we first suppose to be either a reflecting or refracting telescope. If we view, through this system, a distant circular object, of a known diameter, we shall see it double, and in general its two images will be separated from each other. Then we remove it further or bring it nearer until these two images touch each other by their opposite edges. When this takes place, we know that the rays proceeding from the borders of this object, after having traversed the instrument, emerge from it, making with each other an angle precisely equal to C. Now, since we know the diameter M of the object, and its distance which we call Δ, we can easily calculate the visual angle a, under which the same rays cross each other at their

incidence upon the first glass; for $\frac{M}{\Delta}$ expresses its trigonometri- Trig. 30.
cal tangent. The ratio of these angles or $\frac{C}{a}$, will therefore express the magnifying power of the instrument.

Now, in order to determine exactly the angle C, we can view the object through the double prism alone, and carry it farther off or bring it nearer, until its two images appear to touch each other. Then, from the known diameter of the object and its Fig. 135.
distance, we can calculate the visual angle which it subtends, and this will be the value of C. But M. Arago rendered this observation more exact by viewing the two images through a small telescope, before which, in contact with the object-glass, was placed the double prism which gave more distinctness without altering the coincidence of the images. Moreover, instead of a single object, he substituted several of different diameters, at the same distance, and even gave them a triangular form, in order to be able, without displacing them, to choose, in each experiment, the visual angle suited to the double prism, whose amplitude he wished to determine.

212. The method of M. Arago may be applied also to the microscope, under all the different forms of the eye-glass. Only the observation through the instrument, must be made upon an object very near and divided into small portions, like the object-glass micrometer, for example, already described. When we 193.
have placed this micrometer before the object lens, at a conven- Fig. 136.
ient distance for seeing distinctly through the eye-glass, the image the dimensions of which are traced upon it, we place the double prism between the eye-glass and the eye; and directing the double refraction perpendicularly to the series RR of marks traced upon the glass, we count the number RR', of divisions, embraced by the divergence of the two rays. Suppose it equal to m lines. This then will be the real magnitude of the object, which, being seen through the instrument, and brought by it to the distance D, of distinct vision, subtends at the eye the constant angle C; its apparent magnitude, as it is seen through the instrument, will therefore be equal to the distance D, multiplied by the trigonometrical tangent of the angle C; that is, equal to D tang. C; an expression which may be reduced to $\frac{DC}{206265}$, if we sup- Top. 143.

59. pose the angle A converted into seconds. It only remains, then, to divide this apparent magnitude by the real magnitude m of the
61, 62. object, as in the case of a simple magnifying glass, and the quotient $\frac{DC}{206265\,m}$ will express the magnifying power. It is hardly necessary to remark, that the distances D and m must be expressed in units of the same kind.

Instruments employed in Optical Experiments.

213. After having described the instruments which serve to enlarge the power of vision, it is proper to say a word concerning certain other kinds of optical apparatus, remarkable for the beauty or singularity of their effects.

Camera Obscura.

A converging object-glass adjusted to the shutter of a dark room, will concentrate the rays which come from external objects; and if these objects are very distant, compared with the focal distance of the glass, and situated nearly in the direction of its axis, it will give distinct images which may be received upon a white screen. These images are inverted; but in order to render them erect, it is sufficient to bring to the object-glass, instead of the direct light of the object, that of the image, already re-
Fig. 137. flected and inverted by a metallic mirror MM. This apparatus is called a *camera obscura*. We may substitute for the screen a plate of ground glass; and for the room, a box fitted by means
Fig. 138. of a curtain to receive the head. It can then be transported with ease, for the purpose of landscape painting.

Dr Wollaston has remarked, that the best form for the object-glass of a camera obscura, is that of a meniscus convex towards the image, and concave towards the object, as represented in the figure. And some fortunate experiments, made by Cauchoix, seem to indicate that the ratio of curvatures the most favourable, is that of 5 to 8. The shortest of the two curvatures belongs to the surface turned towards the image, because the lens must be converging.

Megascope.

214. Here, as in the preceding case, an object-glass is adjusted to a window shutter; but, instead of causing it to produce the images of distant objects, we place without the room, at a small distance, in the direction of the axis an object strongly illuminated by the light of the sun, either directly or by reflection from several mirrors. If this object is not one of too great dimensions, a distinct image will be formed in the room, the distance and magnitude of which will depend upon the focal distance of the object-glass, and the distance at which the object is placed before it. In proportion, therefore, as we bring the object nearer to the principal focus, we can obtain larger images; but as these will also be thrown at a greater distance, we must be guided in this respect by the dimensions of the room, and must limit ourselves to such distances as will give images sufficiently magnified and at the same time well defined. The images will appear inverted, but may be made erect by inverting the object. Such is the *megascope*. Instead of a single object-glass, we may employ several combined, so as to become achromatic. Then the limits within which the images are distinct, are sufficiently great to enable us to form, in this manner, magnified or reduced representations of pictures and statues, or even of natural figures. M. Charles, who invented this instrument, contrived to magnify objects from 2 to 20 times. In this state, it may be very usefully employed in numerous researches relating to natural philosophy and natural history, where it is necessary to determine with precision the forms and outlines of objects whose smallness or delicate texture prevents their being measured directly. In this case, we receive the image on a plate of ground glass, and sketch the outlines on the opposite surface of the glass or on transparent paper applied to this surface. This process, although graphic, is susceptible of very great accuracy. Fig. 139.

215. The *magic lantern* is simply a portable megascope, in which transparent objects are illuminated by the light of one or several lamps. The term *phantasmagoria*, or the raising of spectres, has been given to the exhibition of an optical apparatus, similar to the magic lantern, in which the distance of the object from the con-

ferences; and he placed the body in the divergent cone formed by this beam, either permitting all the rays to pass, or when he wished to operate with homogeneous light, causing the beam to pass through a glass which transmitted only the red rays. Then, instead of receiving the diffracted stripes upon ground glass, he received them immediately upon the eye, thus preserving much more of the brilliancy. Lastly, in order to join to this mode of observation a means of measurement, he applied to the eye a magnifier fixed upon a firm foot, and adapted to move horizontally, by means of a screw, along a scale divided into equal parts. Thus he was able to bring the axis of this magnifier successively before each bright or dark stripe; and he fixed precisely the position of this stripe, by referring it to a very fine wire extended before the glass exactly at its focus. In this manner he was able to measure upon the scale of equal parts, the intervals of the stripes, by knowing the distance through which the wire passed to apply itself to each of them; and this method is so exact that none of the most minute circumstances of the phenomena can escape observation; and hence nothing was wanting in order to determine the physical laws of these phenomena.

According to a very ingenious remark of Dr Young, the nature and arrangement of the stripes may always be represented with a very near approximation, by supposing that the light which falls upon the borders of the bodies interposed, darts off radiating in all directions from these borders, and interfering either with itself or with the light which continues to be transmitted directly.

Let us take, for example, the case where the body interposed in the luminous cone, is a simple opaque lamina with rectilinear borders. We have seen that in this case two systems of fringes are formed, one exterior with respect to the shade of the lamina, and the other interior. According to Dr Young, the interior are formed by the mutual interference of the luminous beams which proceed from the opposite borders of the plate, by virtue of radiant reflection. Indeed these two portions of light are precisely in the same situation as the two luminous points reflected in the experiment with the mirrors; also the disposition of the interior stripes, whether luminous or obscure, as well as the ratios of their intervals, are absolutely similar. If we conceive the series of points in space where the same kind of interference is produced

at different distances behind the lamina, which gives the successive places where the same stripe appears, we shall find that these points are sensibly in a straight line; and their intervals when measured, are exactly conformable to what the calculation of the interferences indicates.

323. With respect to the exterior stripes, we may consider them as formed by the interference of the light directly transmitted with the light radiated by each border; but here, as in the case of the reflected rings, we must suppose the loss of an interval equal to $\frac{1}{2}\,l$. We thus find that the successive positions in which each stripe ought to appear, for the different distances, are not in the same straight line, but in hyperbolic curves of the second degree; and this is perfectly confirmed by experiment. We must not infer from this, that in diffraction the motion of light is not rectilinear, for it is not the same ray of light which forms the stripe of the same order at different distances; and the rays alone are subjected to rectilinear motion. The change of ray at different distances may be inferred from this simply, that we can view the stripes in space, either with the naked eye or with a magnifier; for it is necessary that the rays which form them should converge and afterwards separate, in order that the magnifier may be able to receive them, and give a sensible image of their point of meeting.

In the case where the diffracted stripes are formed by the passage of a luminous beam between two plates with rectilinear borders, we may with a very near approximation, attribute them to the interference of the two portions of light which fall upon the opposite borders of these plates.

Nevertheless, there are several physical peculiarities of the phenomenon, which are not easily reconciled with this hypothesis. M. Fresnel has even shown that the measures of the stripes are not completely explained when they are taken with extreme precision. He has found that the feeble portion of light, capable of being reflected by the borders of bodies, is not sufficient to produce the intensities of the observed stripes, and that we must suppose the concurrence of rays which pass exteriorly to the sphere of contact of these borders. Accordingly, he considers all the parts of the direct luminous undulation, as so many centres of distinct agitations, the effects of which must extend spherically to all points of space to which they can be propa-

gated; after which, for each of these points the definitive effect must result from the interferences of all the partial agitations which reach it. If this supposition be applied to the free propagation of a spherical wave, in a homogeneous medium, it gives a diminution of light reciprocally proportional to the square of the distance, agreeably to observation. But when a part of the wave is intercepted, it indicates in the different points of space to which it is afterwards propagated, alternations of shade and light, which agree minutely in their disposition and intensity, with what we observe in the diffracted stripes. This principle has enabled M. Fresnel to comprehend, with extraordinary precision, all the cases of diffraction, and to unite them in formulas which express under all circumstances their mutual dependance. But the exposition of these nice results would lead us beyond the design of an elementary work. While we admit all that is established in this case by experiment, it must be allowed that the theoretical deductions from the principle of partial agitations, as explained by M. Fresnel, has been called in question by geometers of the highest authority, particularly by M. Poisson. See *Annales de Chimie et de Physique*, for 1823.

324. I will here add the enunciation of a singular phenomenon, which is easily explained on the principle of interferences. When a narrow and opaque lamina forms interior fringes with respect to its shadow, these fringes may be made to disappear, by placing an opaque screen in contact with one of the borders, or interposing it to a certain degree in the luminous rays, either before or behind the opaque lamina. This phenomenon was discovered by Dr Young. M. Arago has added the circumstance, that the disappearance may be effected in like manner by the approach of a transparent screen sufficiently thick, and that it is progressive according to the thickness; so that thin screens at first only transfer the fringes from the side where they are found. It is remarkable that the quantity of this transference may be calculated according to the laws of interferences, when we know the thickness and refracting power of the plate. If two screens are placed on the two sides of the opaque lamina, the effect is equal to the difference of the transferences which each of them would have produced separately. MM. Arago and Fresnel have made use of this process in measuring the refracting power of gaseous substances, and it has been attended with

a precision of which no other method is capable. See *Annales de Chimie et de Physique*, for 1816 and 1817.

325. The principle of interferences is as yet the only one by which we have been able to explain the peculiarities of diffraction; and in this particular the phenomenon favours the theory of undulations. Nevertheless, if we examine carefully the details of the explanation, we shall find it to be rather a representation of the phenomenon, than a rigorous mechanical theory of it. It were a great desideratum, therefore, to account for this phenomenon upon the principles of the materiality of light, which have hitherto given us such clear ideas and such precise measures as to so many of its other motions. It would be necessary to determine the kind of forces which the particles of bodies exert upon the particles of light, in order to inflect them as they are inflected in the diffracted stripes, and to do this independently of the chemical nature of these bodies, and simply according to the form of the borders which terminate them. But this appears to be very difficult; and it is probable that if such forces exist, they are of a different character from any which is presented by the ordinary phenomena of refraction and reflection.

e instant. The interior of the
rgan, which may be destined
ed in figure 148. It is a kind
brane of the same nature as the
except that the folds, instead of
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ontinuation of the optic nerve itself,
the eye, and applied along its con-
this membrane is oblique to the axis
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ack part of the crystalline, to which
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ids. Naturalists have given it the name
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ush out and draw in the posterior cavity of
manner as to change, by pressure, the curva-
lline, or its distance from the retina, in order to
o the different distances of objects?

e of fishes, represented in figure 149, exhibits dif-
ies. Destined to live in a medium, the refracting
h is nearly equal to that of the aqueous humour in
n, this fluid would not produce any refraction. Ac-
place is supplied by a viscid liquid, probably of a
racting power than pure water, the quantity being very
e same time. By a necessary consequence, the pupil
ely near the cornea, and this circumstance deprives it in
he properties which it possesses in man, but which are
necessary here on account of the small refraction which
experience in entering the humours of the eye from a
nedium. Moreover the pupil of fishes is not susceptible
tion, and the iris is usually tinged on the outside with
lively colours, although on the inside it is always black.
ystalline, applied almost immediately to the back part
cornea, is very nearly spherical. We are absolutely
of the reason why nature has given it this form; but it
to be adapted to vision in the liquid medium in which
e; since, in most birds which dive in the water, we find
talline in like manner spherical. In other respects, it is
d of concentric laminæ like that of other animals.

read by means of the index how slight a movement of the screw is necessary for this change, it will appear truly astonishing.

From what has been said, we may determine whether a glass surface supposed to be plane is really so; for when the screw of the spherometer has been brought just into contact on one part of this surface, we have only to transfer the instrument to the other parts of the glass without touching the screw, and we shall see if the contact still continues with the same precision.

Let us suppose this condition satisfied. If then we slide between the glass plane and the point of the screw, a plate with parallel faces, however thin we suppose it, it is evident that the spherometer will jostle. The quantity by which it is necessary to turn the screw to restore the contact will determine the thickness of the plate interposed. But the pressure of the screw upon the plate in this operation, might break it if it were very thin, or at least might mar its surface if it were susceptible of indentation. To avoid this, the plate must not be inserted directly under the screw. We should first place the screw upon a fragment of plane glass with parallel faces, whose equality of thickness has been previously verified by the spherometer. This fragment being placed upon the principal glass plane, we bring the screw into exact contact with the upper surface, the three other points resting on the principal plane; then we introduce between this last and the fragment, the plate which we wish to measure. After this interposition the spherometer will jostle, and the contact is restored by turning the screw; the distance through which the screw moves, marked by the index, will indicate the thickness sought; and this will be done without the slightest risk, however fragile or thin the plate may be.

II.

Polarisation of Light.

THE polarisation of light is a property discovered by Malus, which consists in certain affections that the rays of light assume on being reflected by polished surfaces, or refracted by these same surfaces, or transmitted through substances possessing double refraction.

Though it would be impossible here to give a complete exposition of the details of these phenomena, we will at least describe some of the experiments by which they may be exhibited.

The first and principal of these consists in giving to light a modification, such that the rays composing a pencil will all escape reflec-

tion when they fall on a reflecting surface under certain circum stances.

As an instance, suppose a beam of solar light *SI* to fall on the first surface *LL* of a plate of glass, smooth but not silvered, making with the surface an angle of 35° 25′; it will be reflected in the direction *II′*, making the angle of reflection equal to that of incidence. Let it then be received on another plate of glass, smooth but unsilvered, like the former; generally speaking it will be again reflected with a partial loss. But the reflection will cease altogether if the second glass be placed like the first, at an angle of 35° 25′ to the line *II′*, provided also it be so turned that the second reflection shall take place in a plane *II′L′* perpendicular to that of the first, *SIL*. Fig. 166.

In order to make this disposition of the glasses more clearly intelligible, we may imagine that *II′* is a vertical line, that *IS* is directed north and south, and *I′L* east and west.

Before we enter upon the inferences to be drawn from this remarkable experiment, I will make a few observations on the manner of performing it.

Many kinds of apparatus may be devised to attain this end. A very convenient one is represented in figure 167. It is very simple, and is sufficient for all experiments on polarisation. It consists of a tube *TT′*, to the ends of which are fixed two collars which turn with sufficient friction to keep them fast in any position. Each of them bears a circular division which marks degrees. From two opposite points of their circumference proceed two brass stems *TV*, *T′V′*, parallel to the axis of the tube, and between them is suspended a brass ring *AA*, which may be turned about an axis *XX* perpendicular to the common direction of the stems. The motion of the ring is likewise measured by a circular graduation, and it may be confined in any position by screws. When a plate of glass is to be exposed to the light, it must be fixed on the surface of the ring; then it may be placed in any situation whatever with respect to the rays of light which pass through the tube; for the collar, turning circularly round the tube, brings the reflecting plane into all possible directions, preserving a constant inclination to the axis, and this inclination may be varied by means of the proper motion of the ring round its axis *XX*. The graduated circle which regulates this motion should mark zero when the plane of the ring is perpendicular to the axis of the tube, and the divisions on the two collars should have their zeros on the same straight line parallel to the axis. In constructing the apparatus one should take care that these conditions are fulfilled; but it is of no great consequence that they be so exactly, as any error may be

compensated by repeating each observation on both sides of the axis, and taking the mean of the numbers of degrees found in the two opposite positions.

If it be desired, for instance, to repeat Malus's experiment described above, a plate of glass must be placed on each ring, and they must be disposed so as to be inclined to the axis at angles of 35° 25′. Then the graduated circle of one of the collars must be brought to mark zero, and the other 90°, that the planes of reflection may be perpendicular to each other. The tube must then be secured, and a candle placed at some distance in such a position that its rays may be reflected by the glass along the axis *TT′*. This will happen when on looking through the tube the reflection of the candle is seen in the first glass. Every thing being thus arranged, the reflected rays will meet the second glass at the same angle of 35° 25′; then according to the different positions given to the collar *T′T′* which carries this glass, the light proceeding from the second reflection will be more or less intense, and there will be two particular positions in which there will be no reflection at all, of those at least which are specularly reflected by the first glass. Care must be taken to put a dark object behind the glass *L′L′* on the side opposite to the reflected light, in order to intercept the extraneous rays which might be sent on this side from exterior objects, and which, passing through the glass, and arriving at the eye, would mix with the reflected rays that are the subject of the observation. The same precaution should be taken for the glass *LL*; and indeed as this is never used except to reflect light at its first surface, the back of it may be blackened once for all with India ink, or smoked with a lamp; it would not do to silver it for a reason that will be given hereafter.

For the light of the candle mentioned above may be substituted that coming from the atmosphere, which may be received into the tube when reflected by the first glass *LL*; but in this case to preserve to the rays the precise inclination required for the phenomenon, the field of the tube should be limited by some diaphragms with very small apertures placed within it. The first glass should be blackened or smoked as before mentioned to intercept any rays that might come by refraction from objects situated under it. In this manner, on looking through the tube, when the glass *LL* is turned towards the sky a small brilliant white speck will be seen, on which all the experiments may be made. The perfect whiteness of this spot is a great advantage; it is an indispensable qualification in many cases, where different tints are to be observed and compared; it is impossible to succeed as well with the flame of a candle or any other

inflamed substance, as none of these flames are perfectly white. Lastly, the brightness of the incident light must be modified, so that the portion irregularly reflected by the two glasses may not be sensible; for this portion, being after such reflection in the state of radiant light, cannot be polarised in one single direction; the other part which is regularly reflected, alone undergoes polarisation, and therefore alone escapes reflection at the second glass.

Whatever be the nature of the apparatus employed, the process will always be the same, and the same phenomena of reflection will be observed on the second glass. To exhibit them in a methodical manner, which will allow us easily to take them all in at one view, we will suppose, as above, that SIL the plane of incidence of the light on the first glass coincides with that of the meridian, and that the reflected ray II' is vertical. Then if the collar $T'T'$ which bears the second glass be turned round, this glass will also turn the reflected ray, making continually the same angle with it, and the second reflection will be directed successively to all the different points of the horizon; this being premised, the phenomena that will be observed are as follows;

When the second or lower glass is so placed that the second reflection shall take place in the plane of the meridian like the first, the intensity of the light, finally reflected, is at its maximum. As this glass is turned round it reflects less and less of the light thrown on it.

Finally, when the lower glass faces the east or west point, all the light passes through it, without being reflected at either surface.

If the collar be turned still farther round, the same phenomena recur in an inverse order, that is, the intensity of the light reflected increases according to the same degrees by which it before diminished, and attains the same maximum state when directed towards the meridian, and so on through the whole circle.

It appears then, that during a whole revolution of the glass the intensity of the reflected light has two *maxima* answering to the *azimuths* 0 and 180°, and two *minima* answering to 90° and 270°. Moreover, the variations are altogether similar on different sides of these positions. These conditions will be completely satisfied by supposing, as Malus does, that the intensity varies as the square of the cosine of the angle between the first and second planes of reflection.

The results of this interesting observation being thus collected into one point of view, we may draw this general conclusion from them, that a ray reflected by the first surface is not reflected by the second (under a particular incidence), when it presents its *east* or *west side*

to the surface, but that in all other positions it is more or less reflected. Now if light be an emission, a ray of light can be nothing else but the rapid succession of a series of particles, and the *sides* of it are only the different sides of these particles. We must therefore necessarily conclude that these have faces endued with different physical properties, and that, in the present case, the first reflection turns towards the same point of space, faces, if not similar, at least endued with similar properties. This arrangement of the particles Malus denominated the ***polarisation of light***, assimulating the operation of the first glass to that of a magnet which turns the poles of a number of needles all in the same direction.

Hitherto we have supposed that the incident and reflected rays made angles of 35° 25′ with the glasses; it is indeed only under that angle that the phenomenon takes place completely. If while the first glass remains fixed, the inclination of the second to the ray be ever so little altered, it will be found that the second reflection will not be entirely destroyed in any position, though it will still be at a minimum in the east and west plane. If again, the inclination of the ray to the second glass being preserved, that on the first be changed, it will be seen that the ray will never pass entirely through the second glass, but the partial reflections which take place at its surfaces are at a minimum in the above mentioned position.

Similar phenomena may be produced by means of most transparent substances besides glass. The two planes of reflection must always be at right angles, but the angle of incidence varies with the substance. According as the refracting power of this is greater or less than that of the ambient medium, the angle of polarisation, measured from the surface, is greater or less than half a right angle. We have seen that for glass this angle is 35° 25′; for sulphate of barytes it is only 32°, and for diamond only 23°. If glass plates be placed in essential oil of turpentine which has a refracting power almost exactly equal to that of glass, the angle of polarisation will be found to differ very little indeed from 45°. The reflection at the second surface is supposed to take place on the ambient medium which bounds the glass. In general, according to an ingenious remark of Dr Brewster, the angle of polarisation is characterized by the reflected ray being perpendicular to the refracted. The angles calculated on this hypothesis agree singularly well with experiment, and also confirm the rule given above for the different magnitudes of these angles, as will easily appear from figures 168, 169, and 170, in which the refracting power is supposed to be respectively greater than unity, equal to unity, and less than unity.

This law applies equally well to substances which, like the diamond and sulphur, never produce more than an incomplete polarisation, for the quantity of light reflected is invariably a minimum for the angle so determined.

If the mode of observation which we have applied to smooth glass plates be universally employed, it may serve to show that polarisation, when complete, is always a modification exactly of the same kind, for all substances; for when a beam of light has been once polarised, it will equally pass through all substances, with the exception mentioned above, provided each be presented to it under its proper angle, and whatever be the nature of the first or second substance employed, the variation of intensity in the light, after the second reflection, is always subject to the same laws.

To represent these circumstances geometrically, let us consider a ray II' polarised by reflection on a glass plate LL, and through any one of the particles composing it, let there be drawn three rectangular axes $c\,z$, $c\,x$, $c\,y$, the first coinciding with the ray, the second in the plane of reflection SIC, the third perpendicular to both the others. Then when the ray II' meets a second glass $L'L'$, placed so as to produce no reflection, the reflecting forces which emanate perpendicularly from the glass, must be perpendicular to the axis $c\,x$; moreover they must act equally on particles lying towards $c\,x$, and $c\,x'$, for if the glass be turned a little from the position of no reflection, the effects are found to be symmetrical on each side of that position. The action, therefore, of these reflecting forces, under this incidence, cannot make the axis $x\,c\,x'$ turn either to the right or left, any more than the force of gravity can turn a horizontal lever with equal arms. They cannot bring the axis into their own plane, in which we see it was in the first reflection, by which the polarisation took place on the glass LL This proves that it is on that axis that the properties of the luminous particles depend. We shall for this reason call it the *axis of polarisation*, and suppose its direction similar and invariably determined for each particle. Further, for the sake of conciseness, we will call $c\,z$ the *axis of translation;* but we do not suppose this invariable in each particle, and we will consider it only as relative to its actual direction, in order to leave the particle at liberty to turn about its axis of polarisation. According to these definitions all the results that we have hitherto obtained may be expressed very simply and clearly in the following manner. Fig. 166.

When a ray of light is reflected by a polished surface, under the angle which produces complete polarisation, the axis of polarisation of every reflected particle is situated in the plane of reflection, and perpendicular to the actual axis of translation of that particle.

If the incident particles are turned so that this condition cannot possibly be fulfilled, they will not be reflected, *at least under the angle of complete polarisation.* That happens when the axis of polarisation of an incident particle is perpendicular to the plane of incidence, the angle of incidence being properly determined *à priori.*

Generally speaking, when a polished surface receives a polarised ray under the angle at which it would itself produce complete polarisation, if it be made to turn round the ray without changing that angle, the quantity of light, reflected in different positions, varies as the square of the cosine of the angle between the plane of incidence, and the axis of polarisation.

When a ray of light has undergone polarisation in a certain direction, by the process above described, it carries that property with it, and preserves it without sensible alteration, when made to pass perpendicularly through even considerable thicknesses of air, water, and, in general, any substance that exerts only single refraction; but doubly refracting media alter, in general, the polarisation of a ray, and in a manner, to all appearance, sudden, communicating to it a new polarisation of the same nature in a different direction. It is only when crystals are held in certain directions, that the ray can escape this disturbing influence. Let us endeavour to compare more closely these two kinds of action.

That of crystals of one axis has been studied by Malus, who has comprised its effects in the following law. When a pencil of light naturally emanating from a luminous body, passes through a crystal of one axis, and is divided into two pencils having different directions, each of these pencils is polarised in one single direction; the ordinary one in the plane passing through its direction and a line parallel to the axis of the crystal, the extraordinary one perpendicularly to a plane similarly situated with respect to its direction. Each of these rays, when received on a plate of glass after its emergence, shows all the characters of polarisation that we have described.

This law obtains equally, when the ray has been polarised by reflection before its passage through the crystal. The two refracted pencils are always polarised, as if they had been composed of direct rays, but their relative intensities differ according to the direction of the primitive polarisation given to them; this direction must therefore have predisposed the particles to undergo from preference one or the other of the refractions.

These two laws were discovered by Malus. The analogy already remarked, between crystals of one axis and those of two, indicates sufficiently how it is to be extended to the latter; to find the direction

of polarisation for the ordinary pencil, draw a plane through its direction, and through each of the axes of the crystal. If either of these axes existed alone, the ordinary pencil would be polarised in the plane belonging to it. Now it is actually found polarised in a plane intermediate between these two, and the extraordinary pencil perpendicularly to the analogous plane drawn through its direction between the two planes containing the axes. If the angle between these be equal to nothing, the crystal has but one axis, and the direction of polarisation is conformable to Malus's indications. This law has been directly verified on the two pencils refracted by topaz; as for other crystals in which it has been possible to verify it directly, we may, by the consideration of some other phenomena that will shortly be mentioned, infer that it applies to them also.

These laws of polarisation are applicable in all cases where the two pencils transmitted by a crystal are observed separately, but when they are received simultaneously, and in nearly the same direction, that of their apparent polarisation is found to be modified, and at the same time their coincidence produces certain colours, which M. Arago first observed, and of which M. Biot determined the experimental laws. The most simple arrangement for exhibiting these colours, is to place a thin lamina of some crystallized substance, in the direction of a white ray, previously polarised by reflection, and to analyze the transmitted light by means of a doubly-refracting prism. The light is thus separated into two portions, the colours of which are complementary to each other, and identical with those of the rings between two glasses. One of these portions appears to have preserved its primitive polarisation, whilst the other exhibits a new polarisation, of which the direction depends on that given to the axes of the crystal by turning the lamina round in its own plane.

Following gradually in this manner the direction of the polarisation given to a particle of light, transmitted through different thicknesses of a crystalline medium, it will be found to undergo periodical alternations, which, if light be an emission, indicate an oscillatory motion of the axes of the particles accompanying their progressive motion. M. Biot has designated this fact by the name of *moveable polarisation*, which is merely the expression of the observed results.

If the system of undulations be adopted, the colours of the two images may be attributed to the interference of the two pencils into which the incident polarised light separates, in passing through the lamina. This is what Dr Young does, and it is remarkable that calculations founded on this principle, gave him the nature of the tints, and the periods after which they recur, precisely as M. Biot had de-

termined them by experiment. As to the alternations of polarisation, they become, in the undulatory system, a compound result produced by the mutual influence of the interfering rays, and it is easy to deduce from observation the conditions to which the mixture of the waves must be subjected to produce the new direction of apparent polarisation. M. Fresnel has done this, and the indications of his formula have been found conformable in all respects to the laws deduced by M. Biot from observation.

These interferences of the rays may be produced without the assistance of crystalline laminæ; we may equally well employ thick plates, provided the rays pass through them at very small inclinations to their crystalline axes. If the experiment be made with a conical pencil of light, large enough to give the various rays composing it inclinations sensibly different to the axes, so that they shall experience double refractions sensibly unequal, these rays, analyzed after they emerge, offer different colours united in the same system of polarisation; and the union of these colours forms round the axes coloured zones, the configuration of which indicates the system of polarising action exerted by the substance under consideration. This kind of experiment is therefore very proper for exhibiting the axes, and indicating the mode of polarisation which any given substance impresses on the rays.

Upon the whole, the interferences of polarised rays offer very remarkable properties, many of which have been discovered and analyzed by MM. Arago and Fresnel with great ingenuity and considerable success, but as the limits of this work do not allow of a full exposition of them, we shall only cite one, which is, that rays polarised at right angles do not affect each other when they are made to interfere, whereas they preserve that power when they are polarised in the same direction. It is not only crystalline bodies that modify polarisation impressed on the rays of light; MM. Malus and Biot found by different experiments, made about the same time, that if a ray be refracted successively by several glass plates placed parallel to each other, it will at length be polarised in one single direction perpendicular to the plane of refraction. Malus by a very ingenious analysis of this phenomenon, has moreover shown that it is progressive, the first glass polarising a small portion of the incident light, the second a part of that which had escaped the action of the first, and so on. M. Arago, measuring the successive intensities by a method of his own invention, has shown that they are exactly equal to the quantity of light polarised in contrary directions at each reflection. A phenomenon analogous to this is produced naturally in

prisms of tourmaline, which appear to be composed of a multitude of smaller prisms, united together, but without any immediate contact. All light passing through one of these prisms perpendicularly, is found to be polarised in a direction perpendicular to the edges, so that if two such prisms be placed at right angles, on looking through them a dark spot is seen where they cross. This property of the tourmaline affords a very convenient method for impressing on a pencil of rays a polarisation in any required direction, and for discovering such polarisation when it exists.

Moreover, M. Biot has discoverd that certain solid bodies, and even certain fluids, possess the faculty of changing progressively polarisation previously impressed on rays passing through them ; and by an analysis of the phenomena produced by these substances, he has shown that the same faculty resides in their smallest particles, so that they preserve it in all states solid, liquid, and aeriform, and even in all combinations into which they may happen to enter. M. Fresnel has found certain analogies between these phenomena and those of double refraction, which seem to connect the two together most intimately through the intermediation of total reflection.

Since reflection and refraction, even of the ordinary kind, modify the polarisation of light, we may expect to find this effect produced when rays of light are made to pass through media of regularly varying density. It is accordingly found that all transparent bodies which are sufficiently elastic to admit of different positions of their particles round a given state of equilibrium, as glass, crystals, animal jellies, horn, &c., produce the phenomena of polarisation, when they are compressed or expanded, or made unequally dense by being considerably heated and then cooled suddenly and unequally. These phenomena, discovered originally by Dr Seebeck, have been since studied and considerably extended by Dr Brewster, who has moreover remarked, that successive reflections of light from metallic plates produced phenomena of colour in which both M. Biot and he recognised all the characters of alternate polarisation.

Knowing, by what precedes, the experimental laws, according to which light is decomposed in crystals endued with double refraction, we may consider these effects as proper means for determining the mode of intimate aggregation of the particles of such bodies, and for affording some insight into the nature of their crystalline structure. Light becomes thus, as it were, a delicate instrument with which we examine the interior of substances, and which, insinuating itself between their minutest parts, permits us to study their arrangement, at which mineralogists previously guessed only by inspection of their

external forms. M. Biot has shown the use of this method, applying it to a numerous class of minerals designated by the general name of *mica*, and he thinks he has decisive reasons for believing that several substances of natures extremely different as to their composition and structure have been improperly comprised under that name. He has also made use of the phenomena of alternate polarisation, to construct an instrument which he calls a *colorigrade*, which, producing in all cases the same series of colours, in exactly the same order, merely by the nature of its construction, affords a mode of designation just as convenient for comparison as that furnished by the thermometer for temperatures.

Many other experiments have been made, and are daily making, and many other properties have been discovered in polarised light; but we have been obliged to confine ourselves to the results, which are, perhaps not the most important part of the subject, although the easiest to explain; our aim in this rapid sketch being rather to stimulate than to satisfy the desire of knowledge on this branch of science which presents so vast a field for research both in theory and experiment, and which, though so lately discovered, has already furnished some useful applications to physics and mineralogy.

III.

The Relations of Light and Heat or Caloric.

In the body of this treatise we have considered only those properties of light which are made known to us by vision. In this single point of view, we have been made acquainted with a variety of physical characteristics belonging to this element. We shall now, for a moment consider the power which light has of heating bodies exposed to its action; and shall endeavour to ascertain whether this phenomenon proceeds from the identity of light and heat, or from the coexistence of both principles in the luminous rays.

We shall first state an important fact discovered by the great astronomer Sir William Herschel. Having proposed to measure the calorific energy of the different rays of the solar spectrum, he placed a very sensible thermometer in each of the seven divisions of colour marked by Newton; he then observed to what degree these thermometers rose in each of them above the point at which they stood in the surrounding air. He thus found that this degree was higher

Proceeding in this manner M. Berard obtained the same results as Sir W. Herschel, with respect to the augmentation of the calorific power from the violet to the red, but he found the maximum of heat at the extremity of the spectrum, and not without it. He fixed it at the point where the ball of the thermometer was just covered by the extreme red rays; he perceived the temperature decrease progressively as the ball of the thermometer reached the shade; and finally placing the thermometer entirely without the visible spectrum, where Sir W. Herschel fixed the maximum of heat, the elevation of the temperature above the surrounding air was only a fifth part of what it was in the extreme red rays. The absolute intensity of the heat produced was also less in the experiments of M. Berard, than in those of Sir W. Herschel. It is impossible to determine whether these differences depended upon the substance of the prisms and the diversity in the apparatus, or upon some other physical circumstance belonging to the phenomenon itself.

M. Berard was desirous of ascertaining whether these properties would manifest themselves separately in each of the portions into which light is divided when it traverses a crystal endued with the property of double refraction. He therefore caused the solar beam to traverse a prism of Iceland spar. The division of the ray formed two spectrums, which exhibited the same properties; in both, the calorific property existed in less intensity within the limits of the spectrum, and was retained beyond the last sensible red rays. Thus, when the ray is divided by traversing the crystal, the calorific power is also divided between the two luminous portions.

In this operation the luminous particles are polarised by the crystal. In order to ascertain whether the *obscure calorific* particles experienced the same effect, M. Berard received the solar beam upon a piece of polished and transparent glass, forming with it an angle of 35° 25′, in order that the reflected portion might be completely polarised. The reflected portion was then received upon another glass forming with it the same angle of 35° 25′, and disposed in such a manner that it could turn conically under this constant incidence. This was precisely the apparatus of two glasses used by Malus, to which we have before had occasion to refer. It is evident that by turning the second glass, two positions would be found where no light was reflected; it only remained then to ascertain whether any heat was here reflected. For this purpose M. Berard placed a concave metallic mirror in such a manner as to collect the rays reflected by this glass and concentrate them upon the bulb of a thermometer placed at its focus. But in order to observe easily the different peri-

ods of the phenomenon, he connected the thermometer closely with the mirror, and the mirror with the glass; so that when the glass turned, the two other pieces turned with it, preserving always the same position with respect to it. Things being thus disposed, M. Berard brought the second glass successively into all possible azimuths about the ray, and he found that in the positions where no light was reflected, no heat was any longer reflected; for the thermometer placed at the focus of the mirror, did not rise when in these positions, although it rose by a very sensibly quantity, when the glass was placed in the azimuths where the reflection of light could take place upon its surface. In this experiment, therefore, as in the preceding one with Iceland spar, the obscure calorific principle accompanies the luminous particles and conforms to their action.

Although in the case in question, it must be admitted that the calorific principle is very little separated from the light itself, since it is free only for a very small space below the extreme red of the spectrum; still, there is a variety of other circumstances, in which we find it in great abundance, almost entirely separated from light. This is the case whenever we observe bodies which give out strong heat, without any sensible appearance of light; such, for example, as a ball of iron heated below the point where it becomes red, and still more vessels of tin or glass filled with boiling water. The sensation of heat which these bodies produce at a distance on our organs, and the effects they produce upon the thermometer cannot be ascribed in general to a transmission by contact, through the medium of the particles of air or vapour which intervene between us and them; for the calorific impression produced by them, is felt as well in a horizontal or downward direction as in any other, which is altogether contrary to the direction in which the particles of air, or vapour, or any other fluid whatever, move when their temperature is raised; the dilatation which they experience causes them to ascend, as has been abundantly proved both from reason and experiment. We must, therefore, conclude that in these and all analogous cases, there is an immediate transmission of obscure caloric, though we cannot absolutely determine whether the caloric is a material principle, radiating like light, or whether it consists only of vibrations propagated through an imponderable medium; two modes, which, if the medium be very elastic, exhibit nearly the same effects. Now, by substituting dark heated bodies in the place of the spectrum before used, M. Berard found that the effects produced upon the thermometer, still took place according to similar laws.

If one of the dark heated bodies of which we have been speaking, be placed before a concave metallic mirror, we find that there is a focus of heat produced by reflection; and this focus is formed at precisely the same point with that of luminous rays proceeding from the same body. The calorific emanation, therefore, is reflected specularly like light, making the angle of reflection equal to the angle of incidence; and as the specular reflection of light is nothing or nearly nothing, from rough bodies, when it does not fall very obliquely upon their surface; so there are certain surfaces which reflect obscure caloric with different degrees of intensity. It is strongly reflected, for example, from the surface of polished metals, but much less from the surface of glass, however perfectly polished. It was for this reason that M. Berard made use of a metallic mirror in the experiment above mentioned.

We have considered only the colorific and calorific properties of light. We shall now speak of its chemical effects. This point also has been accurately investigated by M. Berard. Chemists had long observed that when the muriate of silver and other white salts are exposed to the influence of light, they soon become black. Gum guaiacum, exposed thus to light, passes from yellow to green, as has been observed by Dr Wollaston. MM. Gay-Lussac and Thénard have made known an action of this kind still more immediate and energetic; for, upon exposing to a solar beam a mixture of hydrogen gas and chlorine in equal volumes, a detonation immediately took place, the product of which was hydrochloric acid, heretofore called muriatic acid. M. Berard made use of these different substances as reagents, in examining and ascertaining the chemical properties of the different rays of the spectrum. For having placed in the spaces occupied by the different colours, small pieces of paper impregnated with muriate of silver, or small vessels filled with a mixture of the two gases, he was enabled to judge of the energy of the action of the different rays by the intensity and rapidity with which chemical changes took place in the substances thus exposed. He ascertained in this manner, that the chemical action was really most intense towards the violet extremity of the spectrum, and that they extended as had been maintained by M. Ritter and Dr Wollaston, a little beyond this extremity. Moreover, when these substances were exposed for a certain time to the action of each ray, which could be easily done on account of the immobility of the spectrum, he succeeded in observing sensible effects, though continually decreasing in intensity, in the indigo and blue rays; whence he considered it probable that if reagents still more sensible were employed, analogous effects

might be observed, though more feeble, in the other rays. In order to render evident the great disproportion which exists, in this respect, between the energies of the different rays, M. Berard concentrated, by means of a lens, all that part of the spectrum which is comprised between the green and extreme violet, and he concentrated in a similar manner, by another lens, all that portion which extends from the green to the other extremity. This last portion was collected into one point sensibly white, and so dazzling that the eye could hardly endure it. Nevertheless the muriate of silver remained exposed for more than two hours to this vivid light, without experiencing any sensible alteration. On the contrary, when exposed to the other portion which was much less white and dazzling, in less than ten minutes the muriate was found to become black. M. Berard concluded from this experiment that the chemical effects produced by light are not owing solely to the heat which it developes in bodies, by combining with their substance; since, on this supposition, the power of producing chemical combinations might be expected to be most intense in the rays, which possessed, in the highest degree, the power of producing heat. But perhaps we shall find less opposition between these two considerations, if we bear in mind, that, according to the experiments of De Laroche, there may be essential differences between the obscure caloric employed by chemists, to alter certain combinations, particularly the vegetable colours, and the caloric of the spectrum in the part which does not produce these effects. For example, the difficulty would not exist, if the obscure caloric, obtained by artificial heat, were wholly or partly analogous to the equally obscure emanations which take place at the violet extremity of the spectrum, and there does not seem to be any impossibility in such a supposition.

These experiments of M. Berard leave no doubt that the different portions of a solar ray, dispersed by the prism, have very different properties as to the power which they possess of producing vision, heat, and chemical combination. Are we then to attribute these three powers to three distinct kinds of rays, existing independently of each other, and capable each of producing only one single effect. If this be the case, it will also be necessary that each of these species should be capable of being separated by the prism into an infinity of different modifications, like light itself, since we find by experiment that each of the three properties, chemical, illuminating, and calorific, is distributed, though in very unequal proportions, over a certain extent of the spectrum. Thus, according to this hypothesis, we must conceive three spectrums, one calorific, one chemical, and one luminous, superposed upon each other. It must likewise be admitted that

each of the substances which compose the spectrums, and even each of the particles of unequal refrangibility which compose these substances, is endued, like the particles of visible light, with the property of being polarised by reflection, and of escaping the influence of the reflecting force, as is observed with respect to the luminous particles, and in the same cases. A like analogy must exist also in the other properties. But instead of this complicated system, we suppose, conformably to the phenomena, that the solar light is composed of an assemblage of rays unequally refrangible, and consequently capable of being differently modified by the action of bodies, which supposition requires that there should be original differences, either in their masses, their velocities, or their affinities. Why should these rays which differ in so many respects, all produce upon thermometers and upon our organs, the same effects as to heat and light? Why should they have the same power in forming and separating chemical combinations? Is it not very natural to suppose that vision may take place in our eyes only between certain limits of refrangibility, and that too great or too small a refrangibility may render the rays equally unfit to produce this effect. It is possible that these rays are capable of exciting a vision in other eyes; perhaps they are capable of exciting it in certain animals; on this supposition all that is marvellous in the above phenomena disappears, or rather it becomes a part of the general action of light. In a word, we may suppose the calorific and chemical properties to vary throughout the whole extent of the spectrum with the refrangibility, but according to different functions, in such a manner that the calorific property shall be at its *minimum* at the violet extremity of the spectrum, and at its *maximum* at the red extremity; while, on the contrary the chemical faculty, expressed by another function, shall have its minimum at the red extremity, and its maximum at the violet extremity or a little beyond it. This single supposition, which is the simplest representation of the phenomena, perfectly satisfies all the facts stated above, and even enables us to predict a great number of them from analogy alone. Indeed, if all the rays which produce vision, heat, and chemical combinations, are equally rays of light, they must all necessarily be reflected from polished bodies, and reflected according to the same law, making the angle of reflection equal to the angle of incidence; whence it follows that they will be, in like manner, concentrated or dispersed by concave or convex mirrors. They must, moreover, be all polarised in traversing a crystal endued with the property of double refraction, or in being reflected from glass, ice, &c., under a determinate incidence; and when they have received these modifications, they must

be reflected from another surface of the same kind, if it be placed in such a manner as to render its reflecting force efficacious upon the luminous particles. On the contrary, if this force produces no effect upon the visible luminous particles, invisible light will no longer be reflected; for the cause which determines or prevents reflection, appears to act equally upon all the particles, whatever be their refrangibility; and must therefore operate upon invisible light, since the condition of visibility or invisibility has reference only to the constitution of our eyes, and not to the nature of the particles themselves which produce the sensation in us. Finally, since, according to the observations of De Laroche, obscure caloric, emanating from bodies gradually heated, approximates gradually to the conditions and properties belonging to luminous caloric, we might suppose that when the emanation begins to become visible, it would be immediately analogous to the least calorific part of the spectrum, which is the extremity of the violet. Accordingly we observe that flame of whatever kind, in its incipient state, is violet or blue, and only attains to whiteness when it has reached a high degree of intensity. Still this view of the subject from the very circumstance that it supposes a progressive state, is not inconsistent with the idea of particular properties, belonging exclusively to a particular stage of the progression. Thus the calorific emanations of different temperatures, and the luminous emanations of different colours, may differ from each other in the property of producing vision, heat, and chemical action, in the power of being transmitted through transparent substances, and perhaps in many other particulars which we have not yet discovered.

IV.

Measure of the Intensities of Light.

It often happens in optical researches, that we have occasion to compare the intensities of two lights presented at the same time or successively. In the first case, which is the most simple, we illuminate separately with these two lights, equal discs of very white paper, or some other unpolished body which is a good reflector; then, viewing at the same time the two discs, we remove the most intense of the two lights, until they appear equally bright. Hence the intensities will be as the squares of the distances of the lights from the discs. This partial illumination may be obtained by illuminating a white space with these two lights, and interposing before them a

small opaque disc, the shades of which will indicate the points separately illuminated. It is sufficient, then, to render these shades equally deep, by varying the distance of the luminous body from the screen. We may also admit the two lights separately through Fig. 180. two conical tubes united at their vertices, and terminated at this place by two equal discs of white paper. Then viewing both discs at once, having the head covered for the purpose of excluding all foreign light, we give notice to an assistant which of the two luminous objects must be removed or brought nearer, until the two discs appear equally bright. When we have arrived at this result, the intensities of the two lights are proportional to the squares of the distances from their respective discs.

But if we wish to compare two lights which are not visible at the same time, we have only to select a third, whose brightness is of such a nature as to sustain itself without variation, and compare it successively with each of the other two. By employing these processes and various others of a similar kind, Bouguer obtained a variety of curious results from which the following are selected.

Table of the quantities of light reflected by the surface of water under different obliquities.

1000 expresses the number of incident rays.							
Obliquities reckoned from the surface.	Number of reflected rays.	Ob.	No.	Ob.	No.	Ob.	No.
0° 30′	721	5° 0	501	17° 30′	173	50°	22
1 00	692	7 30	409	20	145	60	19
1 30	669	10	333	25	97	70	18
2 00	639	12 30	271	30	65	80	18
2 30	614	15	211	40	34	90	18

Table of the quantities of light reflected by the first surface of polished glass.

Obliquities of incidence reckoned from the surface.	Number of rays reflected.	Obliq.	No.	Obliq.	No.
2° 30′	584	15°	299	50°	34
5	543	20	222	60	27
7 30	474	25	157	70	25
10	412	30	112	80	25
12 30	356	40	57	90	25

Table of the quantities of light reflected by black marble polished.

Angle of the incident rays.	Number of rays reflected.
3° 35′	600
15 0	156
30 0	51
80 0	23

The first reflection under the angle of 3° 35′ is nearly as intense from marble as from mercury. The same may be said with respect to all plane bodies of whatever substance. They all become very good reflectors, when the incident rays make very small angles with the surface. But the reflecting force diminishes very rapidly as the direction of incidence approaches to a perpendicular. In this they differ from bodies whose reflecting force is powerful; for in these, the intensity of the reflected light undergoes only slight variations, under different incidences. In the case of mercury and telescope metal, for example, the whole extent of this variation scarcely amounts to $\frac{1}{7}$ or $\frac{1}{8}$ from 0 to 90°. According to the experiments of Bouguer, for an incidence of 21° reckoned from the surface, mercury reflects about 637 rays out of 1000. Consequently, reflection under all other angles may vary from about 700 to about 600. This metal, which is perhaps the best of all reflecting substances, absorbs then more than one quarter of the light which falls upon it; and this absorption is much greater in bodies which reflect more imperfectly.

V.

Description of the Kaleidoscope.

'This instrument, in its simplest form, consists of two reflecting planes, made of glass or metal, from 5 to 10 or 12 inches long, and about an inch broad. These reflectors being put together with two of their edges in contact, and their reflecting faces inclined, for instance, at an angle of 60°, or the sixth part of a circle, when they are put in a tube, and the eye is placed at *E*, as near the angular point as possible, it will observe the opening *AOB* multiplied six times, and arranged round the centre *O*. Fig. 182.

'If any object, however shapeless, is placed before the opening *AOB*, and near the ends of the reflectors, the eye at *E* will observe

in the two adjoining sectors an inverted image of this irregular object, apparently facing the direct image in *AOB*, and the direct and inverted image will form an object perfectly symmetrical; and these symmetrical images being multiplied by successive reflections, the whole circular space, composed of six sectors, will present to the eye a most perfect picture.

'If the object placed in *AOB* consist of pieces of coloured glass, lying in a cell bounded by discs of glass, the pictures increase in beauty, and as the glass fragments change their position by the motion of the cell, a succession of the most perfect pictures will be displayed, which are literally infinite both in number and variety.

'The objects which give the finest outlines by inversion are those which have a curvilineal form, such as circles, ellipses, looped curves like the letter *S*, spirals, and other forms. Glass, both spun and twisted, and of all colours and shades of colours, should be formed into the preceding shapes, and when these are mixed with pieces of flat coloured glass, blue vitriol, native sulphur, yellow orpiment, differently-coloured fluids moving in small enclosed vessels of glass, &c., they will make the finest transparent objects for the kaleidoscope. A very fine effect is produced when only two colours are used, viz. those that harmonize with each other, such as red and green, blue, and gold yellow. Pieces of glass of these two colours may be mixed with twisted pieces of colourless glass with great effect.

'In the simple kaleidoscope the two reflectors may be fixed at a constant angle, so as to be an aliquot part of a circle, or 360°, or they may be made to vary their inclination by various simple contrivances.

'If the eye is raised above *E*, or if the objects are placed at any distance from the ends of the reflectors, the symmetry of the picture and the uniformity of the light vanish, so that it is essentially necessary to the production of forms perfectly beautiful and symmetrical that the eye be placed as near as possible to the angular point, and the objects as near as possible to the ends of the reflectors.

'The utility of the kaleidoscope may be greatly extended by the addition of a lens. "In considering how this change might be effect-
Fig. 183. ed, it occured to me," says Dr Brewster, "that if *MN* were a distant object, either opaque or transparent, it might be introduced into the picture by placing a lens *LL*, at such a distance before the aperture *AOB*, that its image may be distinctly formed upon the plane passing through *AOB*. By submitting this idea to experiment, I found it to answer my most sangine expectations. The image formed by the lens at *AOB* became a new object, as it were, and

was multiplied and arranged by successive reflections, in the very same manner as if the object *MN* had been reduced in the ratio of *ML* to *LA*, and placed close to the aperture.

' "In the compound kaleidoscope, thus constructed, the furniture of a room, books, and papers lying on a table, pictures on the wall, a blazing fire, the moving branches and foliage of trees and shrubs, bunches of flowers, horses and cattle in a park, carriages in motion, the currents of a river, moving insects, and in short every object in nature may be introduced by the aid of the lens into the figures created by the instrument. When the flames of a blazing fire constitute the object, the kaleidoscope creates from it the most magical fire-works, in which the currents of a flame which compose the picture may be turned into every possible direction.

' "The theory of this instrument; the various forms of annular, parallel, and polycentric kaleidoscopes; its application to the magic lanthorn and the solar microscope; and the mode of employing it in the fine and useful arts, have been explained at great length in my *Treatise on the Kaleidoscope.*" '—*Brewster's Edition of Ferguson's Mechanics.*

VI.

New Optical Experiments, and the Improvement of Glass.

' M. Frauenhofer has been long known on the continent of Europe as a very distinguished practical optician. He has succeeded beyond any other artist in producing flint glass for optical purposes, of the most complete transparency and freedom from flaws and defects. This superiority in his glass has enabled him to prosecute some very important researches. His primary object was to determine with great exactness, for the formation of achromatic object-glasses, the dispersive powers of different species of glass. He first tried the effect of correcting the colour by opposing prisms, viewed through a telescope, which is in fact the same method as that originally proposed by Dr Brewster. But it became an object of attention to examine the dispersion of each coloured ray separately. To do this is a problem which has always been attended with the essential difficulty of not being able to fix upon rays in the spectrum which are strictly homogeneous, and which can at all times be identified with certainty. In order to get over this difficulty, M. Frauenhofer tried, without success, different coloured media and flames; to trials of this kind we shall soon have occasion to allude, as lead-

ing to some important discoveries. Our artist, however, next adopted a plan which he considered as successful; this was to place six lamps in a row behind a small aperture, close before which was a prism. The separate spectrums of each lamp were thus thrown, so that the prism under trial, which was placed at nearly seven hundred feet distance, received only the red rays, for example, from one lamp, and the blue from another, &c., by which means the colours appeared in the form of distinct spaces, separated entirely from each other. We cannot help feeling some difficulty as to the application of this method, but perhaps the description itself is not the clearest that might be given. We do not feel sure that the rays were strictly homogeneous; however, they were capable of exact identification from this further contrivance: a narrow aperture was made in the screen above the six lamps, through which the light of another lamp passed, and was received on the second prism; in viewing this, a bright line was seen at the limits of the red and yellow spaces; this was exactly defined, and by means of its invariable position, in comparison with the coloured spaces below, the observer could always be assured that the same identical ray fell on his prism. A number of measurements were thus made with great exactness, from which the great differences in the ratios of refraction for the same ray in different media, are clearly ascertained.

'But the most important point was the appearance of the bright line above mentioned. This M. Frauenhofer next proceeded to study; he found it exhibited alike by the light from all flames, &c., when received through a narrow aperture. He next tried the light of the sun; this was received into a dark room through a narrow crevice, at the distance of twenty-four feet, by a prism of excellent flint glass; in looking at the spectrum thus formed through a small telescope, he observed not only the bright line before spoken of, but an infinity of lines, some dark and some bright, crossing every part of the spectrum at right angles to the direction of its elongation, and not forming the boundaries of the different coloured spaces, but existing in the middle of them, and in fact, distributed in some places more plentifully than in others along the whole length, in some parts more conspicuous, and in others more faint. Of all these lines the observer has given an accurate delineation; he counted upon the whole 574 of them. If the aperture be so wide as to subtend an angle of more than 15″ to the eye, the lines disappear. Some of the fainter ones also are not seen, unless the eye be shaded from the glare of the brighter parts. With English flint glass, M. Frauenhofer could only see the brightest lines; but with every sort of glass of his own manufacture, and with prisms formed of liquids,

they were all distinctly seen. He then proceeded, by an extended series of measurements, with a repeating circle, to determine the angles of deviation which these lines formed when viewed through different media. These lines in fact supply the great desideratum in researches of this nature, and enabled him to determine the deviations belonging to points in the spectrum strictly definite, with any degree of accuracy.

'From observing the great number of lines crossing the spectrum, we might be led to suppose that the inflection of light at the edges of the aperture had some connexion with the phenomenon; in order to examine this point, M. Frauenhofer varied the experiment in the following manner; he received the rays through a small circular hole nearly 15″ in diameter; the spectrum thus formed had almost no breadth, but in order to widen it, M. Frauenhofer made the rays pass through a semi-cylinder of glass. By this means the length and order of colours remained unaltered, but the breadth being magnified, he saw as before all the lines. By means of the same contrivance he detected similar lines in the light of the planet Venus, without employing any aperture; the brightest lines only were visible, but they coincided in position with the corresponding ones in the solar spectrum. The light of some of the principal fixed stars was subjected to the same examination; in some of these lines were observed in positions different from those before observed. The electric light was tried in the same way; the points of two conductors were connected by a fine fibre of glass, along which the succession of sparks was so rapid as to produce the appearance of a fine line of light. In the spectrum formed by this light, (without passing any aperture,) lines different from any of the former were observed. The light of several was similarly examined, and several curious results obtained.

'Such is a brief outline of the most important parts of M. Frauenhofer's experiments; they indicate a very remarkable property of light, and present appearances which we believe have not yet been accounted for on any known principles. We must here take occasion to remind our readers, that the discovery of the fact itself, (though evidently unknown to M. Frauenhofer,) was made some years ago by Dr Wollaston. His experiments was, however, somewhat different; and owing to the great superiority of his glass, M. Frauenhofer has the merit of having ascertained the almost infinite number of those lines, which in Dr Wollaston's experiments appeared only a few.

'M. Frauenhofer must also have the credit of being the first to apply these lines to the purpose of accurate determination of the

dispersive power, although Dr. Wollaston made a few observations of this kind. It may be satisfactory to many to mention here, that with an ordinary prism of English glass, the principal lines may be very well seen by looking through the prism at a narrow aperture in a shutter or screen placed against a window so as to receive the light of the clouds; this was Dr Wollaston's method; his experiments are given in the Philosophical Transactions, for 1802; he examined also the light from flame. If the blue part of a candle flame be received through a narrow slit, the separation of the colours is very wide and complete.

The mere inspection of the prismatic colours is sufficient to show that the different parts of the spectrum, independently of their colour, possess very different degrees of brightness or illuminating intensity. The late Sir W. Herschel was, we believe, the first who attempted any accurate determination of these relative intensities; he found the greatest illumination in the yellowish green space, and a gradual decrease from thence towards each extremity. M. Frauenhofer tried similar experiments by a different method, and his determinations were made with greater attention to exactness than perhaps any former; but there appear to us two essential difficulties in his method. In the first place, the intensity of each coloured ray was to be equalised with the white or yellowish light reflected by a plane mirror from a lamp; M. Frauenhofer considers it easy, with a little practice, for the eye to judge of this equalisation with the requisite accuracy. This, we must confess, appears to us very doubtful; though the sensation of colour and of intensity may possibly depend on modifications of the same cause, yet the two sensations follow such very different laws, and that difference is dependent upon principles so wholly unknown to us, that we can hardly conceive the possibility of abstracting so entirely from the idea of colour that of intensity, as to enable the mind to decide in any thing like a certain and satisfactory manner upon the equality of illuminating effect in lights of two different colours simultaneously presented to the eye.

'Another and more serious difficulty appears to us to arise from the following considerations; supposing the illuminating intensities to be really equal, it is well established, that if two rays of light, one of a colour approaching more to whiteness than the other, be presented in juxta-position to the eye, the deeper colour of the one will be diluted by the proximity of the lighter colour of the other; that is to say, though not actually combined or blended together, the sensation which the one produces in the eye tends to diminish that which arises from the other. If this, as is highly pro-

bable, is owing to the different convergency required for the two it will obviously take place in a greater degree, in proportion as the coloured ray differs in refrangibility from the white.

'Whatever weight may be attributed to the objections against this particular method, it is certain that the illuminating intensity sustains a regular decrease from the central yellowish green to the violet on one side, and the red on the other. The series of numbers given by M. Frauenhofer decrease in a more rapid ratio than those found by any other observers, and the tendency of the causes just considered as influencing his results, would be precisely that of producing this rapid diminution. But the decrease of illuminating power towards the red boundary, will become a point of considerable interest in the sequel.

'M. Frauenhofer's observations on the illuminating powers of the prismatic rays, led him to several suggestions of practical importance in the construction of telescopes. He attends particularly to the distinction between diminishing the aberration of colour, and producing greater distinctness in the image; as also to the aberration from the want of achromatism in the human eye. When different specimens of glass were examined by the accurate test of the spectral lines, the difference in their dispersive powers was shown, when not otherwise capable of detection. M. Frauenhofer found differences of this kind in specimens taken not only from the same crucible, but from the opposite parts of the same piece of glass. By unwearied diligence and laborious trials, he has, however, at length succeeded in the manufacture of flint glass, to such a degree, that in a crucible containing four hundred pounds, two pieces, one taken from the bottom and the other from the top of the same mass, exhibited absolutely the same power.

'This becomes the place for noticing the results obtained by a fellow-labourer in the same work. M. Guinand was the son of a joiner at Neufchatel; as a youth he worked at that trade; subsequently made watch cases; and thus acquiring some idea of casting metals, undertook, on examining a reflecting telescope, to make one; in which he soon succeeded, without any knowledge of optics, and left entirely to his own resources for every part of the work. His next attempt was to make a pair of spectacles. He learned the art of grinding and polishing the lenses by having once witnessed the process. He hence proceeded to make lenses of telescopes, and constructed several small refracting ones. He now accidentally became acquainted with the principle of the achromatic object glass; and all his energies and labours seemed concentrated upon the means of endeavouring to procure glass free from imperfections for

this purpose. This is in fact one of the most difficult problems with which the practical optician is concerned; and the patience, the sagacity, the perseverence, which M. Guinand displayed, in a long series of attempts under the most discouraging circumstances, to obtain his object, were truly surprising. At every failure he seemed to be occupied solely in studying the cause which had occasioned it. And thus, step by step, he contrived to approach at length towards the wished for object, and produced glass more free from striæ and imperfections than any before made. Every disappointment taught him some further improvement, and it was thus that he acquired, what is perhaps the distinguishing characteristic of his method, the mode of joining together into one large disk separate pieces of glass, selected as the most perfectly homogeneous. These he contrived to soften and unite together again, after which they were formed into the required lens, without any perceptible joining, or imperfection; in this way he has formed lenses of twelve or eighteen inches diameter. In 1805, his fame had reached M. Frauenhofer, who invited him to Batavia, to give his important services to the establishment at Benedictbauern, where glass for optical purposes is largely manufactured under M. Frauenhofer's direction. The glass made by M. Guinand has since become known over Europe; specimens have been tried by the opticians and astronomers of France and England. The report of that eminent artist, Mr. Tulley, as to its great superiority over any made in England, is couched in the strongest terms; and there can be little doubt that, owing to the very perfect transparency which it possesses, we may expect a great increase in the power of refracting telescopes, hitherto so much limited in their degree of improvement. M. Guinand returned to his native place, and continued the construction of telescopes with uncommon ingenuity and success, himself not only having melted, formed, and polished the glasses, and calculated the adjustments, but also constructed every part of the apparatus, and put it together. This remarkable example of untaught genius died 1823, aged seventy-six. His secret is confined to his son, who undertakes to continue the manufacture so important to the scientific world, upon the same principles as the father.

'We before mentioned that M. Frauenhofer's first attempts were directed to obtaining homogeneous light by means of flames and coloured media; in this he was unsuccessful. Dr Brewster, however, and Mr Herschel have been more fortunate.

'Dr Brewster was in want of homogeneous light, to illuminate objects under microscopic examination. Mr Herschel wished to obtain it for the prosecution of certain optical researches. Dr Brew-

ster, after numerous trials, ascertained the remarkable fact, that almost all bodies in which the combustion is imperfect, such as paper, linen, &c., gave a light in which strictly homogeneous yellow rays predominated; that the yellow light increased with the *humidity* of these bodies; and that a great proportion of the same light was generated when various flames were urged mechanically with a blow-pipe, or a pair of bellows. He thence concludes, that the yellow rays are the produce of an imperfect combustion. However, the most important circumstance was that the presence of aqueous vapour increased the quantity of yellow light; this was a new fact, and supplied Dr Brewster with a lamp whose light was truly homogeneous. Diluted alcohol is the pabulum he employs, and he has suggested a convenient form for a lamp for the purpose wanted.

'Various media, such as coloured glasses, were also tried. Dr. Brewster investigated the effect of heat in changing the tints of these glasses; in some, the power of absorbing particular colours is altered transiently, in others permanently. He tried the effect of different media in absorbing the different rays of the spectrum, and has given dilineations of the spectrum, as seen through different coloured glasses.

'In Mr Herschel's experiments the object was nearly the same in the first instance, but he has pursued it in a somewhat different manner from Dr Brewster, and has arrived at some other results of considerable consequence.

'He first examined, as also Dr Brewster did, the effects of certain coloured glasses in almost obliterating certain coloured spaces in the spectrum, whilst others were transmitted in all their brilliancy. This fact was noticed by Dr Young. Mr Herschel, in applying to the examination of it the uncommon powers of his analytical skill, has resolved the phenomena into their most general expression, and thus traced the causes of many interesting consequences which otherwise would not have been deduced.

'For example, one of the glasses he tried was of a ruby red colour; this permitted to pass almost the whole red, and a considerable portion of the orange; and even in strong lights a portion of yellow or a trace of green, but the rest were obliterated. He represents the effect by conceiving a straight line divided according to the proportions of the coloured spaces, to be taken as the abscissa, and at each point ordinates erected, representing the proportion of rays transmitted by any medium; the extremities of these ordinates give a curve, which he calls *the type* of this medium. The nature of this curve is determined by observation for each medium; but Mr Herschel has given an analytical expression, showing the law by which

the nature of the curve is altered, according to an increase of thickness in the medium; this is in fact one of the most curious parts of the subject.

'"It would appear at first sight," Mr Herschel observes, "that the effect of doubling or tripling the thickness of any coloured medium, would simply be to increase the depth and intensity of the tint, but not to alter its character. If a white object appear blue through a blue glass, we should expect it to appear still bluer through two, and yet more so through three such glasses. The above formula shows, however, that this is so far from being the case, that the tint of the emergent pencil is essentially dependent on the thickness of the medium; and that it is only from a knowledge of the relative values of the ratios of the intensity after traversing a thickness equal to unity, for the various parts of the spectrum, that we can say *à priori*, whether the tint of a thick glass will retain any similarity to that of a thin one of the same kind."

'The fact is, the quantity of any coloured ray, transmitted by an homogeneous medium, decreases in *geometrical* progression, as the thickness increases in *arithmetical.* Thus, however trifling the difference may be at first in the effect of two media, it is always possible to render it sensible by taking a sufficiently great thickness; thus the water of the lake of Geneva is indigo-blue, that of the lake of Como, emerald-green, when viewed through a considerable thickness, though colourless in small quantities. Of this, numerous other instances will occur; such as the difference in the colour of the sea according to its depth, so well known to pilots, as often enabling them to perceive their approach to shoals, &c.

'"In some instances the curve has two unequal maxima in different parts of the spectrum; and if at the same time the greater of these should happen to correspond to a ray of feebler illuminating power than the less, the tint, in small thicknesses of the medium, will, generally speaking, be that of the lesser maximum; the greater vividness of these rays giving them a predominance over the others, though more numerous; but as this inequality of number increases with the increase of thickness, the feebler rays will at length begin to influence the tint, and finally obtain the predominance; thus producing in several cases, a complete change of colour, not a little surprising to those who are ignorant of its cause. Dr Thomson's muriated liquor (chloride of sulphur,) which is yellowish green in very small thicknesses, and bright red in considerable ones, is a case in point; a solution of sap-green presents the same phenomenon yet more strikingly. If enclosed between glass plates, slightly inclined, so as form a thin wedge, its colour towards the edge will

all the extremities of bodies, that limit the medium through which light is transmitted. Accordingly, if we introduce a beam of simple light into a dark room, and receive it at a considerable distance upon ground glass, we observe that the extremities of all the bodies near which this light passes, are bordered with luminous fringes, similar to those produced by the two plates when very distant from each other, and exposed perpendicularly to the luminous ray. This last condition is necessary to the precise enunciation of the phenomenon; for we may observe very large fringes between two distant edges, if we incline them considerably to the incident light; and, for the same reason, similar fringes are also formed by reflection, when the incident ray grazes the borders of surfaces under a very great obliquity.

322. But if the single body which forms fringes by its interposition, has dimensions so small that the beam of light can completely envelope it, projecting over the opposite sides, the simultaneous influence of these two edges produces new phenomena, which become sensible at less distances, according as the intercepted portion of light is narrower. For the sake of distinctness, suppose that the body is a small opaque rectangular lamina, only a few hundredths of an inch in width. Then, besides the exterior fringes formed on the parts situated towards the light, as when the body is of an indefinite size, other fringes appear within the shade. When the incident light is simple, these fringes are of the same colour with that; and are separated from one another by intervals absolutely black. Their number is always odd, so that there is always one at the centre of the shade. If we receive them first at very small distances, and then gradually at greater, they are at first very fine and almost imperceptible; then they gradually extend and become more distinct, and their intervals enlarge at the same time. If we receive them upon ground glass where we can observe their formation, we recognise in them the preceding peculiarities; but the observation is rendered very much more precise and distinct, if we employ the different arrangements invented by M. Fresnel, and which apply with equal advantage to all the experiments on diffraction. After having fixed the direction of the luminous beam by receiving it from a heliostat, M. Fresnel concentrated it by a strong magnifier, in a radiating point, almost mathematical, as in the fundamental experiment on inter-

'"The species of light alluded to is remarkable; first, for its perfect homogeneity, and secondly, for its position in the spectrum. When the solar spectrnm, received on a white paper in a darkened room, is viewed through a moderate thickness (0,08 inch) of that glass, cemented to any red glass of a tolerably pure colour, it will be seen reduced to a perfectly circular and well defined image of a deep red colour. If a pin be now stuck in the *centre* of the red circle it will be found on removing the glass from the eye, to have been fixed in what an ordinary observer would call the very *furthest termination* of the red rays; and a mark similarly made at its circumference, will appear to lie wholly without the spectrum, among the dispersed light which usually hangs about its edges; in other words, the red, thus insulated, is of too feeble an illuminating power to affect the sight in the immediate vicinity of the other more brilliant rays, and only becomes visible when they are extinguished, or greatly enfeebled. To an eye defended by such a glass, vision, through a prism with the largest refracting angle, is as sharp, and the outlines of minute objects as free from nebulosity and indistinctness, as if their rays had suffered no refraction. These characters,—the absolute homogeneity of the rays,—their situation precisely at the least refracted limit of the spectrum, and the facility with which they may be insulated, render them of peculiar importance as standards of comparison in optical experiments."

'In this simple and unpretending manner does Mr Herschel announce, what we must consider one of the greatest accessions to the catalogue of optical facts, which has been made since Newton first pointed out the unequal refrangibility of the primary rays. To their number Mr Herschel has added another, whose existence had not previously been suspected; in the analysis of light he has detected a new ingredient, and has thus found a new and exact means of measuring the dispersive powers of different media. To this purpose he has, at the conclusion of his paper, applied the insulation of these extreme red rays, and of the extreme violet; the deviation thus obtained, being of course greater for every sort of glass than any obtained by former methods, and the measurement extremely exact, from the circumstance of the rays being precisely defined and truly homogeneous. The method of operating is, we believe, new, and very simple.

'The utility of the extreme red rays for this purpose is unquestionably very great; but the fact will be interesting to philosophers in a variety of other points of view. We have already made some remarks on the decrease of illuminating intensity in the different spaces of the spectrum, from the centre to the extremities; this

is closely connected with the existence of invisible rays. It has been ascertained that the eye is somewhat deficient in its power of converging red light; from this cause alone, if the red rays were presented to it in an insulated state, the outer part of the red would be indistinct, and it would be very probable that certain extreme rays might exist which would be altogether invisible; but when the rays are presented in juxta-position, the influence of the central rays which converge at a shorter distance will tend to increase the deficiency in the perception of the extreme red; and this would be the case on the supposition that all the rays possessed an intrinsic equal illuminating power, and were all of equal density; but if in this respect they differ, (as we have seen they do,) the diminution will be still more considerable. It would thus be evident, that at whatever distance from the central point the real termination of the spectrum were situated, the apparent illuminating powers must decrease by a much more rapid law, than the absolute and intrinsic intensities would do; so that the apparent limit of the spectrum would be at a much shorter distance from the point of maximum illumination.

'The discovery of the new red rays has, as might be expected excited great interest; they have been recently examined by Mr Powell, who has measured their deviation, and observed them also, in the moon's light. In forming the spectrum as in Dr Wollaston's experiment above described, their appearance is remarkably distinct; in the spectrum of the blue part of a flame they do not exist, although there is much of the more refrangible red."—*British Critic.*

VII.

On the Magnetizing Power of the more refrangible rays of Light.

'Dr Morichini, a respectable physician in Rome, discovered this remarkable property of the violet rays. Professor Playfair saw the experiment by Dr Carpi, in the absence of Morichini, before a party of English and Italian gentlemen. The following account of the experiment was drawn up from a conversation which the writer of this notice had with that distinguished philosopher, and was afterwards submitted to him for his approbation.

"The violet light was obtained in the usual manner, by means of a common prism, and was collected into a focus by a lens of a suffi-

cient size. The needle was made of soft wire, and was found, upon trial, to possess neither polarity nor any power of attracting iron filings. It was fixed horizontally upon a support, by means of wax, and in such a direction as to cut the magnetic meridian at right angles. The focus of violet rays was carried slowly along the needle, proceeding from the centre towards one of the extremities, care being taken never to go back in the same direction, and never to touch the other half of the needle. At the end of half an hour, after the needle was exposed to the action of the violet rays, it was carefully examined, and it had acquired neither polarity, nor any force of attraction; but after continuing the operation twenty-five minutes longer, when it was taken off and placed on its point, it traversed with great alacrity, and settled in the direction of the magnetical meridian, with the end over which the rays had passed turned towards the north. It also attracted and suspended a fringe of iron filings. The extremity of the needle that was exposed to the action of the violet rays, repelled the north pole of a compass needle. This effect was so distinctly marked, as to leave no doubt in the minds of any who were present, that the needle had received its magnetism from the action of the violet rays."

'Such was the state of this subject when Mrs Somerville directed to it her attention; and it is no slight praise to say, that she has set to rest a question on which the scientific world was divided, and that by the sagacity and ingenuity with which she has conducted her experiments, she has rendered visible, even in our northern climate, one of the most delicate of the magnetic influences, which, it was agreed on all hands, required for its developement the serene sky of an Italian climate.

'The following is a general outline of these interesting experiments.

'Having obtained the prismatic spectrum by means of an equiangular prism of flint glass placed in a hole in the window-shutter, Mrs Somerville took a sewing needle, about an inch long, and entirely devoid of magnetism.* Conceiving that no polarity would be superinduced if the whole needle were exposed to its action, she covered one half of it with paper, and exposed the other half to the violet rays of the spectrum cast upon a pannel at the distance of five feet. *In about two hours, the needle had acquired magnetism, the ex-*

* 'This was ascertained by its attracting indifferently either pole of a sewing needle magnetized in the usual way. This magnetized needle was pushed through a piece of cork, in which was inserted a glass cap, and it was in that state made to revolve freely on the point of another sewing needle.'

posed end exhibiting north polarity. This experiment was often repeated, and always with the same result.

'By a similar process, Mrs Somerville ascertained that the *indigo* rays had nearly as great an effect as the violet, and that the *blue* and *green* rays likewise produced the same effect, though in a less degree.

'Mrs Somerville next tried the *yellow*, *orange*, and *red* rays, but neither in them nor in the calorific rays, was the slightest effect produced, even when the experiments were continued for three successive days.

'Mrs Somerville now applied the same method to pieces of clock and watch springs, about 1½ inches long, and from ⅛ to ¼ of an inch broad,* and they were found to receive a stronger degree of magnetism from the violet rays, an effect which was attributed to their blue colour, and their greater extent of surface. Bodkins were not affected. When the violet ray was concentrated by a lens, the magnetic influence was imparted to the needles in a shorter time.

'In order to give additional confirmation to these results, Mrs Somerville exposed unmagnetized needles, half covered as formerly, to the sun's rays transmitted through glass coloured blue by cobalt, and they were distinctly magnetized as before. Needles exposed under green glass received the same property.

'Mrs Somerville now enclosed unmagnetized needles in pieces of *blue* and *green* ribband, one half of each being covered with paper, and after they had hung a day in the sun's rays behind a pane of glass, they had acquired magnetic polarity, the exposed ends being north poles, as in the former experiments. When *red*, *orange*, or *yellow* ribband was used, no magnetic influence was imparted.

'In performing these experiments, Mrs Somerville found that the most favourable time of the day was from ten to one o'clock; and that, as the season advanced, the magnetism acquired was less permanent, as the needle required a longer exposure to acquire the same degree of magnetic virtue.'—*Edinburgh Journal of Science for* 1826.

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* 'When these possessed any magnetism, it was removed by heating.'

VIII.

Of the Colours of Striated Surfaces.

It is well known that beautiful colours are produced upon polished surfaces by means of fine lines or scratches. 'Boyle appears to have been the first that observed these phenomena. Newton has not noticed them. Mazéas and Mr Brougham have made some experiments on the subject, yet without deriving any satisfactory conclusion. But all the varieties of these colours are very easily deduced from the theory undulations.

'Let there be, in a given plane, two reflecting points very near each other, and let the plane be so situated that the reflected image of a luminous object seen in it may appear to coincide with the points; then it is obvious that the length of the incident and reflected ray, taken together, is equal with respect to both points, considering them as capable of reflecting in all directions. Let one of the points be now depressed below the given plane; then the whole path of the light, reflected from it, will be lengthened by a line which is to the depression of the point as twice the cosine of incidence to the
Fig. 181. radius.

'If, therefore, equal undulations of given dimensions be reflected from two points situated near enough to appear to the eye but as one, wherever this line is equal to half the breadth of a whole undulation, the reflection from the depressed point will so interfere with the reflection from the fixed point, that the progressive motion of the one will coincide with the retrograde motion of the other, and they will both be destroyed; but, when this line is equal to the whole breadth of an undulation, the effect will be doubled; and when to a breadth and a half, again destroyed; and thus for a considerable number of alternations; and if the reflected undulations be of different kinds, they will be variously affected, according to their proportions to the various lengths of the line which is the difference between the lengths of their two paths, and which may be denominated the interval of retardation.

'In order that the effect may be the more perceptible, a number of pairs of points must be united into two parallel lines; and, if several such pairs of lines be placed near each other, they will facilitate the observation. If one of the lines be made to revolve round the other, as an axis, the depression below the given plane will be as the sine of the inclination; and while the eye and luminous object re-

tion when they fall on a reflecting surface under certain circum stances.

As an instance, suppose a beam of solar light *SI* to fall on the first surface *LL* of a plate of glass, smooth but not silvered, making with the surface an angle of 35° 25′; it will be reflected in the direction *II*′, making the angle of reflection equal to that of incidence. Let it then be received on another plate of glass, smooth but unsilvered, like the former; generally speaking it will be again reflected with a partial loss. But the reflection will cease altogether if the second glass be placed like the first, at an angle of 35° 25′ to the line *II*′, provided also it be so turned that the second reflection shall take place in a plane *I*′*I*′*L*′ perpendicular to that of the first, *SIL*. Fig. 166.

In order to make this disposition of the glasses more clearly intelligible, we may imagine that *II*′ is a vertical line, that *IS* is directed north and south, and *I*′*L* east and west.

Before we enter upon the inferences to be drawn from this remarkable experiment, I will make a few observations on the manner of performing it.

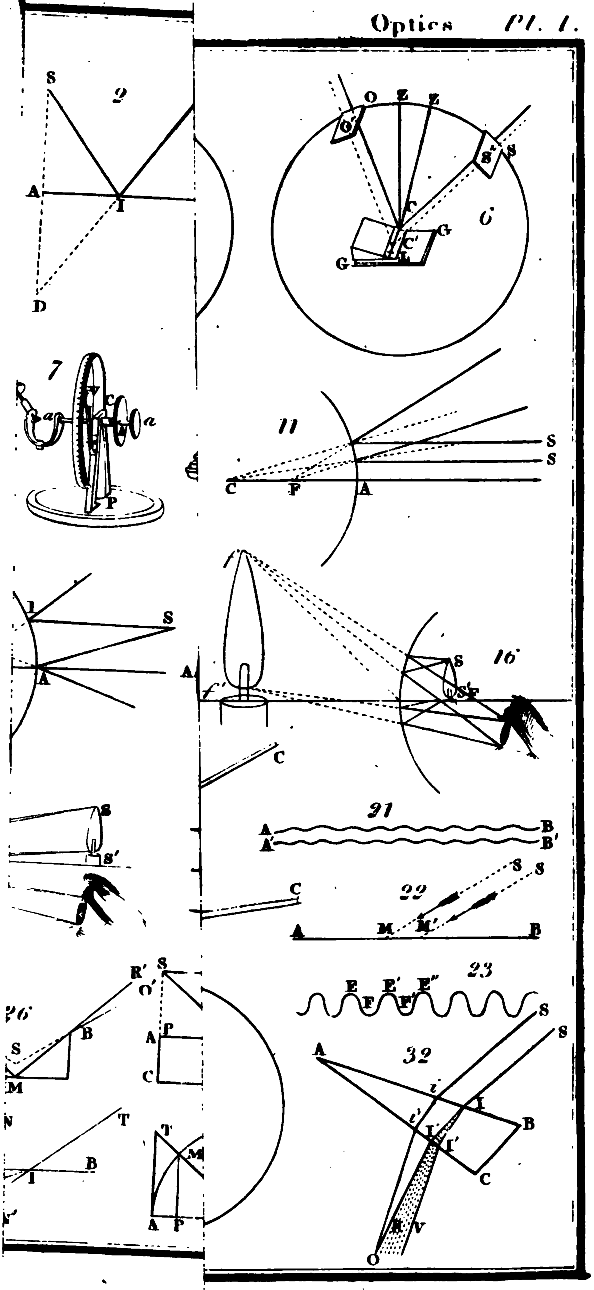
Many kinds of apparatus may be devised to attain this end. A very convenient one is represented in figure 167. It is very simple, and is sufficient for all experiments on polarisation. It consists of a tube *TT*′, to the ends of which are fixed two collars which turn with sufficient friction to keep them fast in any position. Each of them bears a circular division which marks degrees. From two opposite points of their circumference proceed two brass stems *TV*, *T*′*V*′, parallel to the axis of the tube, and between them is suspended a brass ring *AA*, which may be turned about an axis *XX* perpendicular to the common direction of the stems. The motion of the ring is likewise measured by a circular graduation, and it may be confined in any position by screws. When a plate of glass is to be exposed to the light, it must be fixed on the surface of the ring; then it may be placed in any situation whatever with respect to the rays of light which pass through the tube; for the collar, turning circularly round the tube, brings the reflecting plane into all possible directions, preserving a constant inclination to the axis, and this inclination may be varied by means of the proper motion of the ring round its axis *XX*. The graduated circle which regulates this motion should mark zero when the plane of the ring is perpendicular to the axis of the tube, and the divisions on the two collars should have their zeros on the same straight line parallel to the axis. In constructing the apparatus one should take care that these conditions are fulfilled; but it is of no great consequence that they be so exactly, as any error may be

compensated by repeating each observation on both sides of the axis, and taking the mean of the numbers of degrees found in the two opposite positions.

If it be desired, for instance, to repeat Malus's experiment described above, a plate of glass must be placed on each ring, and they must be disposed so as to be inclined to the axis at angles of 35° 25′. Then the graduated circle of one of the collars must be brought to mark zero, and the other 90°, that the planes of reflection may be perpendicular to each other. The tube must then be secured, and a candle placed at some distance in such a position that its rays may be reflected by the glass along the axis TT'. This will happen when on looking through the tube the reflection of the candle is seen in the first glass. Every thing being thus arranged, the reflected rays will meet the second glass at the same angle of 35° 25′; then according to the different positions given to the collar $T'T'$ which carries this glass, the light proceeding from the second reflection will be more or less intense, and there will be two particular positions in which there will be no reflection at all, of those at least which are specularly reflected by the first glass. Care must be taken to put a dark object behind the glass $L'L'$ on the side opposite to the reflected light, in order to intercept the extraneous rays which might be sent on this side from exterior objects, and which, passing through the glass, and arriving at the eye, would mix with the reflected rays that are the subject of the observation. The same precaution should be taken for the glass LL; and indeed as this is never used except to reflect light at its first surface, the back of it may be blackened once for all with India ink, or smoked with a lamp; it would not do to silver it for a reason that will be given hereafter.

For the light of the candle mentioned above may be substituted that coming from the atmosphere, which may be received into the tube when reflected by the first glass LL; but in this case to preserve to the rays the precise inclination required for the phenomenon, the field of the tube should be limited by some diaphragms with very small apertures placed within it. The first glass should be blackened or smoked as before mentioned to intercept any rays that might come by refraction from objects situated under it. In this manner, on looking through the tube, when the glass LL is turned towards the sky a small brilliant white speck will be seen, on which all the experiments may be made. The perfect whiteness of this spot is a great advantage; it is an indispensable qualification in many cases, where different tints are to be observed and compared; it is impossible to succeed as well with the flame of a candle or any other

Optics Pl. I.

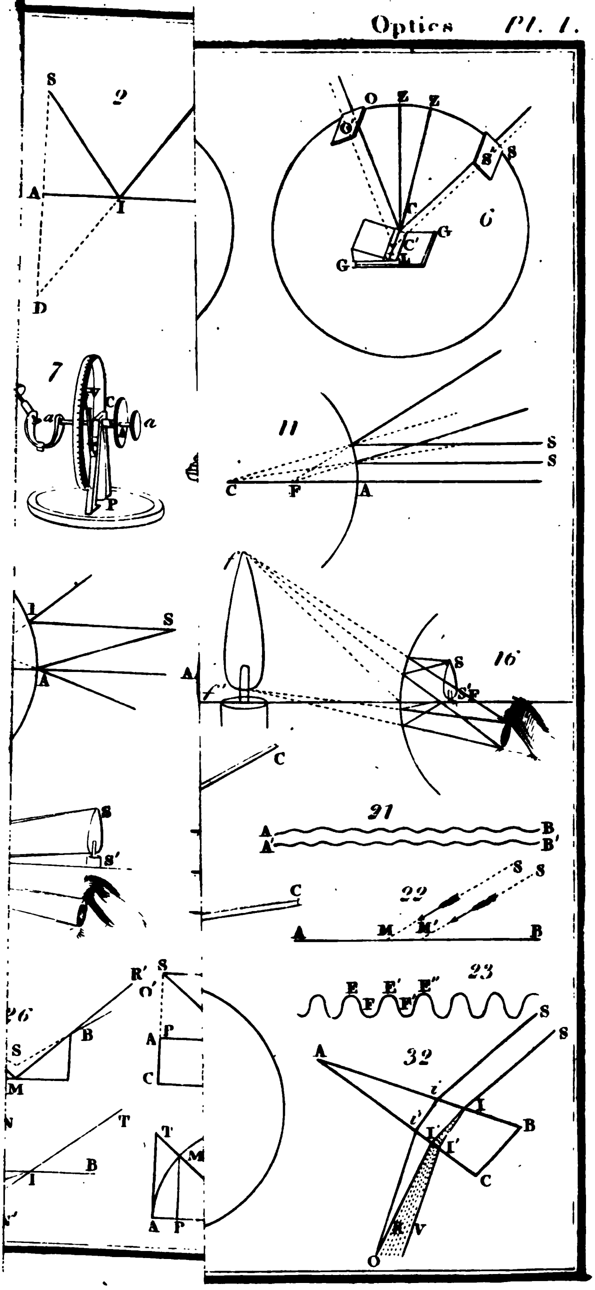


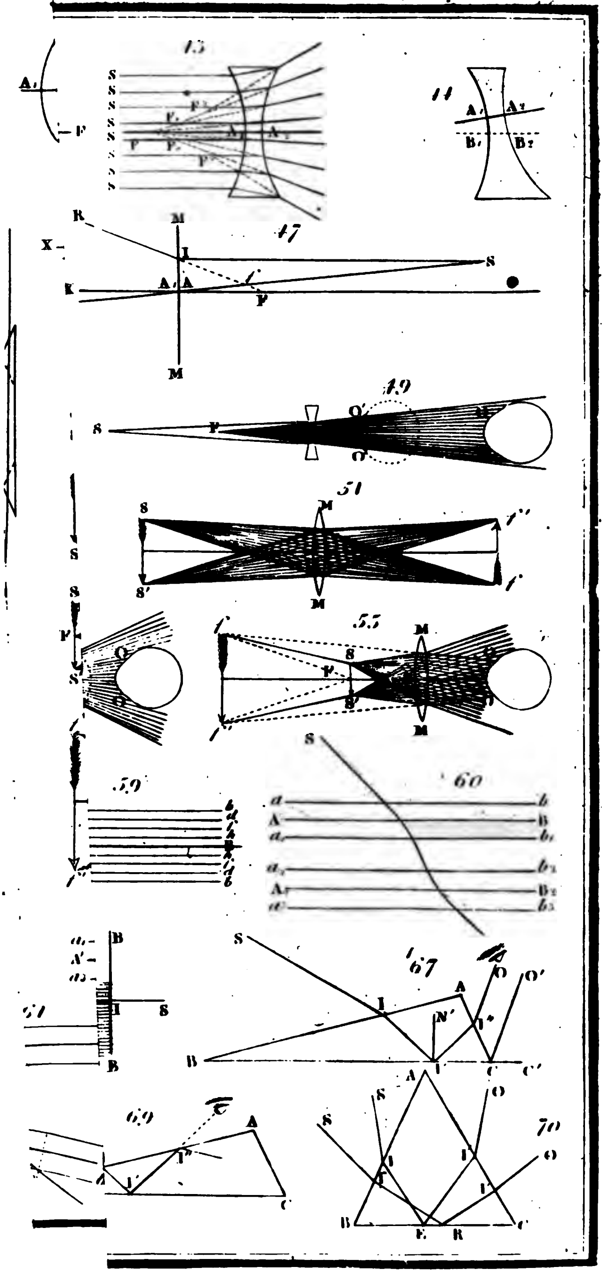
termined them by experiment. As to the alternations of polarisation, they become, in the undulatory system, a compound result produced by the mutual influence of the interfering rays, and it is easy to deduce from observation the conditions to which the mixture of the waves must be subjected to produce the new direction of apparent polarisation. M. Fresnel has done this, and the indications of his formula have been found conformable in all respects to the laws deduced by M. Biot from observation.

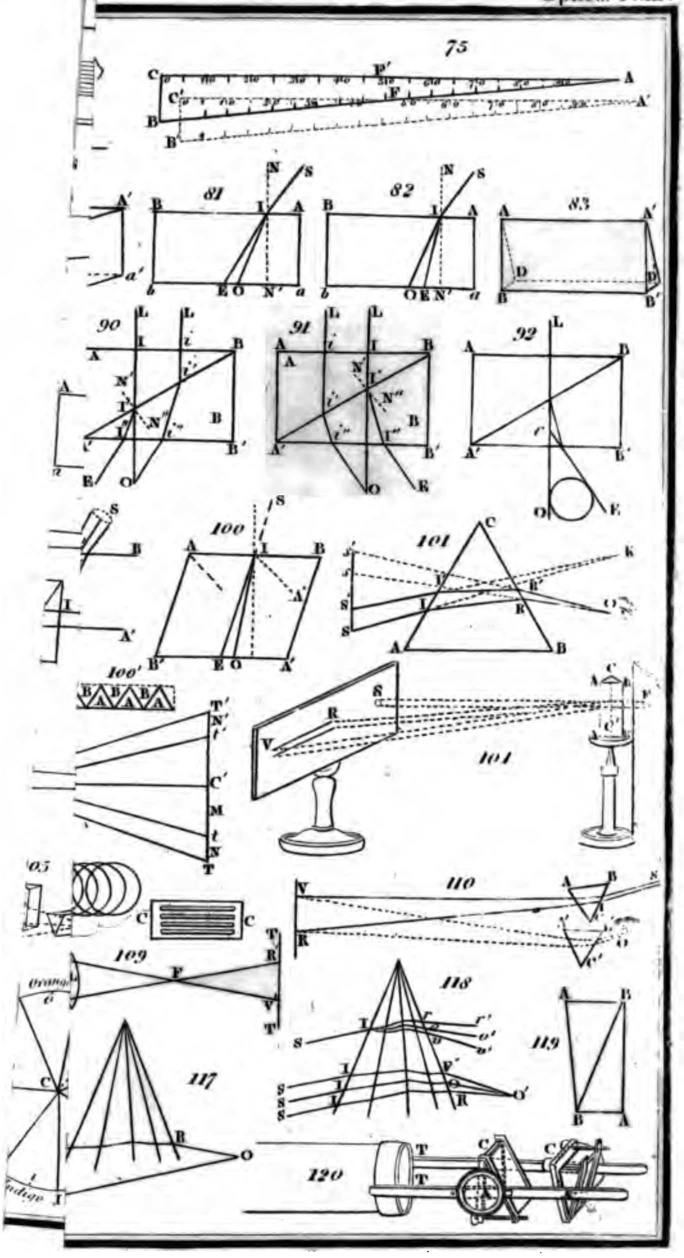
These interferences of the rays may be produced without the assistance of crystalline laminæ; we may equally well employ thick plates, provided the rays pass through them at very small inclinations to their crystalline axes. If the experiment be made with a conical pencil of light, large enough to give the various rays composing it inclinations sensibly different to the axes, so that they shall experience double refractions sensibly unequal, these rays, analyzed after they emerge, offer different colours united in the same system of polarisation; and the union of these colours forms round the axes coloured zones, the configuration of which indicates the system of polarising action exerted by the substance under consideration. This kind of experiment is therefore very proper for exhibiting the axes, and indicating the mode of polarisation which any given substance impresses on the rays.

Upon the whole, the interferences of polarised rays offer very remarkable properties, many of which have been discovered and analyzed by MM. Arago and Fresnel with great ingenuity and considerable success, but as the limits of this work do not allow of a full exposition of them, we shall only cite one, which is, that rays polarised at right angles do not affect each other when they are made to interfere, whereas they preserve that power when they are polarised in the same direction. It is not only crystalline bodies that modify polarisation impressed on the rays of light; MM. Malus and Biot found by different experiments, made about the same time, that if a ray be refracted successively by several glass plates placed parallel to each other, it will at length be polarised in one single direction perpendicular to the plane of refraction. Malus by a very ingenious analysis of this phenomenon, has moreover shown that it is progressive, the first glass polarising a small portion of the incident light, the second a part of that which had escaped the action of the first, and so on. M. Arago, measuring the successive intensities by a method of his own invention, has shown that they are exactly equal to the quantity of light polarised in contrary directions at each reflection. A phenomenon analogous to this is produced naturally in

Optics Pl. I.







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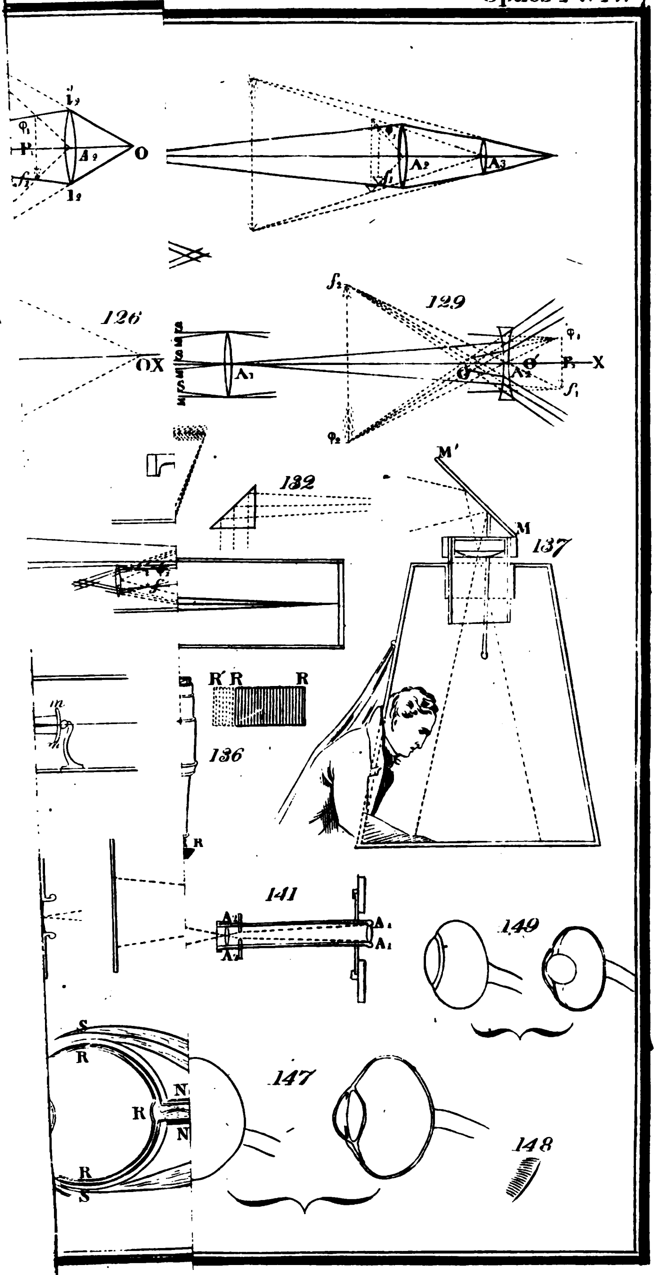

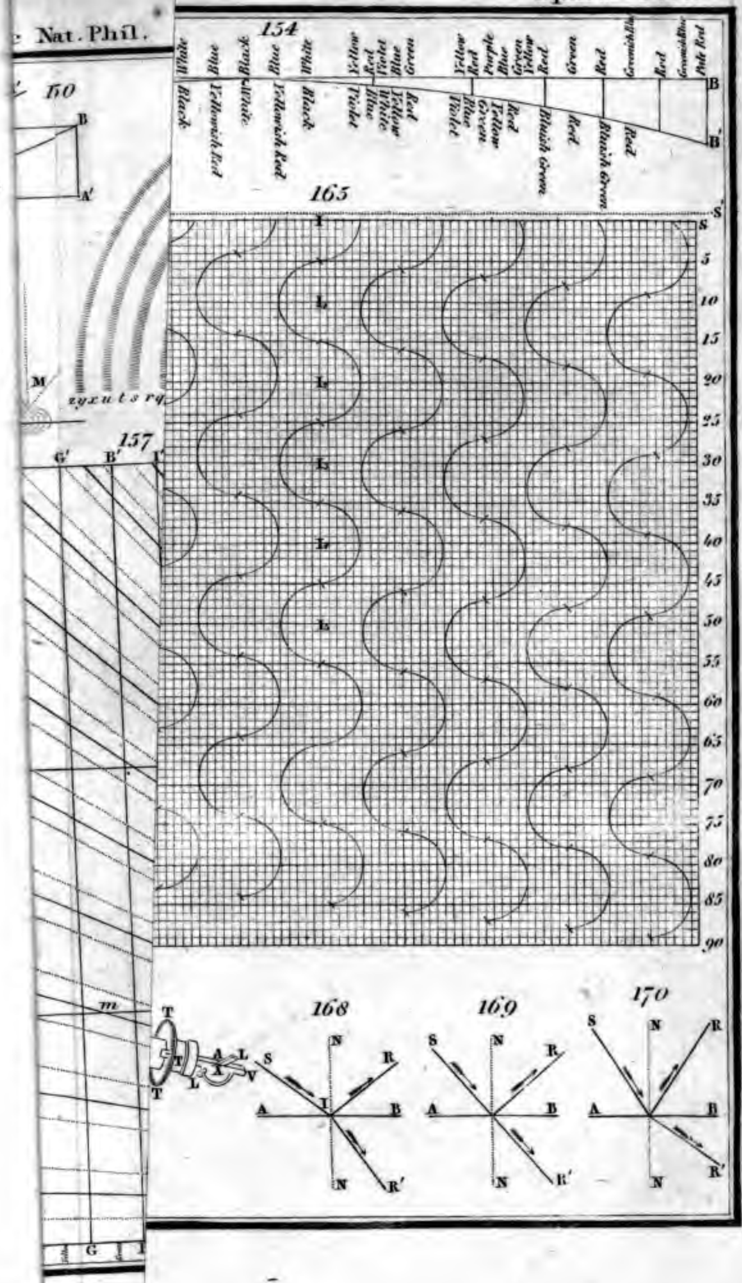
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